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# SCHOOL SCIENCE AND MATHEMATICS

APRIL 1960

# School Science and Mathematics

*A Journal for All Science and Mathematics Teachers*

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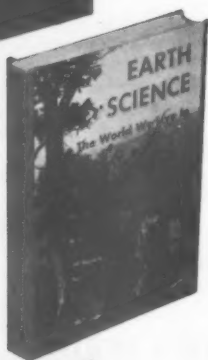
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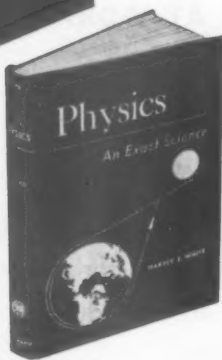
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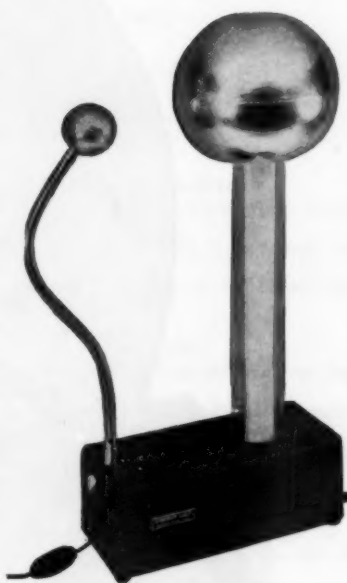
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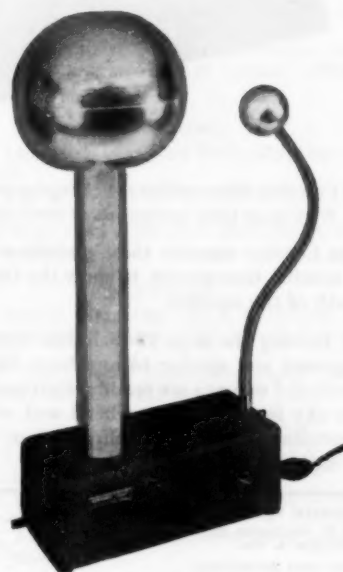
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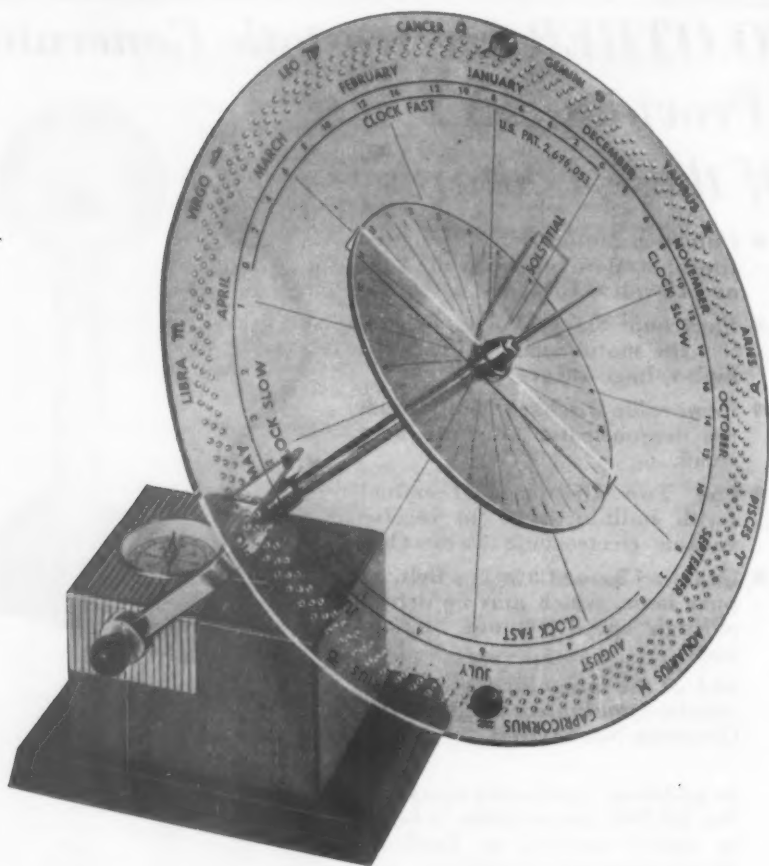
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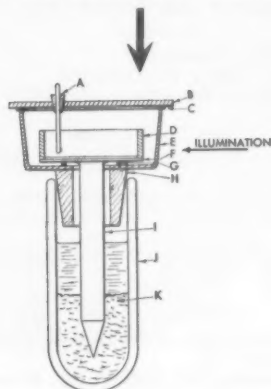
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## Recent Developments in Space Research\*

A. H. Shapley

*National Bureau of Standards, Boulder, Colorado, Member, Space  
Science Board, National Academy of Sciences*

We are only on the threshold of the Age of Space. Extensive though our knowledge may be of the vast reaches beyond the earth's atmosphere, it represents, at best, but a scratch on the great mass of information waiting to be discovered and applied to the benefit of all mankind.

The International Geophysical Year may be best remembered, I think, for the fact that it supplied the key which opened the door to space. I do not mean by this to say that space research dates back only to July 1, 1957. Ever since man began to ponder the mysteries of the sun, the moon and the stars, he has strived to learn more of the regions of space outside the earth. It has been an exceedingly difficult task since the earth's atmosphere, which makes it possible for us humans to exist on our small planet, also hides from us much that is of interest.

Despite ingenious and sensitive instruments, even today the earth-bound scientist is peering into space through three very small windows. Through one window he can look with the naked eye or with optical instruments, but atmospheric turbulence limits the effective resolution of his telescopes to about one-twentieth of the theoretically obtainable value. Through two other windows he can "see" electromagnetic radiations of high energy and short wave length, or low energy and long wave length. But these cosmic radiation and radio

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\* Presented at a joint conference of the Aeronautical Engineering, Mechanical Engineering, and Physics Divisions of ASEE at its 1959 Annual Meeting in Pittsburgh.



windows are in a sense opaque. A large part of the radiations are prevented from reaching the windows at all by the earth's atmosphere and magnetic field. Indeed, atmospheric attenuation limits the earthbound observer to the use of about 20 of the 60 octaves in the electromagnetic spectrum above 100 kilocycles.

Scientific research, thus restricted, leaves many vital scientific problems relating to the earth's environment and the Universe beyond—outside of man's capability to examine critically.

Direct access to the outer atmosphere for scientific observation and experiment has come only in the last dozen years. It was in 1946 that man began to put instruments in vehicles which went above 100,000 feet, when U. S. scientists started using German V2 rockets for scientific research at White Sands. Rocket exploration of the upper atmosphere and scientific looks at outer space from above most of the earth's atmosphere, continued at a slow but rather steady rate, principally in this country, until under the stimulus of the IGY, science from rockets blossomed both here and abroad. By putting together several existing rocket systems, almost with scotch tape, the USSR and U. S. put into orbit during IGY instrumented artificial earth satellites. With other combinations, the first space probes have been sent into interplanetary space.

For the first time, magnetic and gravitational fields, radiations of all sorts, and particles in interplanetary space, and in the vicinity of the earth, the moon and the nearer planets, have all become accessible to direct observation. With the means now at hand, it has become possible not only to sample matter in space but also to observe and perhaps to obtain samples of the material of the moon and the planets.

It is not surprising that the IGY assured its place in history by marking man's first entry into space, since a large portion of the IGY program was focused upon the earth's cosmic environs. Nearly half of the 11 disciplines studied during the IGY were concentrated in the field of upper atmosphere physics—solar activity, cosmic rays, geomagnetism, aurora and airglow, and ionospheric physics. More than three hundred research rockets, eight satellites (with three still to be launched), and four deep space probes have enabled scientists to place scientific laboratories above the earth's masking atmosphere and to make, directly in space, vital observations which are implementing all IGY studies—even those concerned with the earth's interior.

Today we are in the "tin lizzie" stage of space research. Nevertheless, the research carried on to date gives direct promise of the great ultimate power of sounding rockets, satellites and space probes. The scientific results of the first primitive experiments have been revolu-



tionary in advancing our knowledge of the earth and its environment, and they have led to reclassification and re-evaluation of our earlier speculations and hypotheses.

New and valuable data have been obtained on atmospheric temperature, density and pressure. Prior to the IGY, scientists had deduced from scattered rocket experiments that density and pressure of the high atmosphere over White Sands, New Mexico, remained the same from day to night and from one season to the next. Rocket firings during the IGY, however, showed that at *high* altitudes over Fort Churchill, Canada, there is a difference in these measurements between day and night and between summer and winter. But at *low* altitudes the situation was like that of White Sands. So these measurements, we now know, vary geographically and also according to altitude.

From analysis of the decay of satellite orbits, we learned that the atmosphere at satellite altitudes is at least ten times more dense than the best scientific evidence previously indicated, and this in turn implies that the temperature at the highest part of our atmosphere is higher than previously thought possible. We also know that there are large changes every few days in the atmosphere at about 250 miles which can be interpreted as density changes, and these seem closely related to solar activity.

The first direct measurements in space of meteoritic debris have been made. The first experiments indicate that a spherical vehicle in space with a diameter of one yard and a skin thickness of one-quarter inch would not be punctured by a meteorite, on the average, in from 300 to 150,000 years. These figures reflect a wide degree of uncertainty, it is true. Nevertheless, it seems clear that meteoritic debris does not represent a significant space travel hazard. Vanguard I, for example, travelled over 131 million miles during its first year of operation, without apparent damage from this source.

Important new knowledge on the shape of the earth has also been deduced from analysis of satellite orbits. The bulge of the equator seems to be about 500 feet more than previously thought, and there is a north-south asymmetry, very small, which has led people to describe the earth as "pear-shaped."

Information of great practical value has been learned concerning control of temperature in a space vehicle or laboratory. Solar batteries have been proved to be a reliable continuing source of power, even though so far in the milliwatt range. We have increased our understanding of the ionosphere. Great advances have been made in techniques of rocket astronomy.

I could go on, the list is long. Instead, I will dwell for a moment on perhaps the most interesting discovery of the IGY, the Van Allen

radiation belts. As you know, Dr. Van Allen and his colleagues at the State University of Iowa conducted experiments aboard Explorer satellites and Pioneer space probes—both part of the IGY program—which revealed the existence of two belts of extremely intense radiation around the earth. The first belt begins about 1300 miles above the surface of the earth and extends to about 3400 miles. The outer belt begins at about 8000 miles and extends to about 52,000 miles from the earth's surface.

The radiation in the outer zone is thought to consist of charged particles of hydrogen gas ejected by the sun, which stream through space and are trapped in the earth's magnetic field. It has been suggested that the radiation in the inner zone is caused by decay products of neutrons emerging from the earth's atmosphere.

The discovery of the radiation belt has implications far beyond the practical ones which are inevitably emphasized in the press and the popular science journals—who seem preoccupied with the dangers to future astronauts. Knowledge of the existence of radiation trapped in the earth's magnetic field has given new challenges to the theorists who have been trying to understand how particles from outside come into the earth's atmosphere and produce a wide variety of effects—the northern lights, the magnetic storms, anomalies in the ionosphere, and so on. The recipe for previous theories was—1 part observed data, 49 parts mathematics and 50 parts ignorance or speculation. The proportion of this last ingredient is now significantly reduced. We now see the possibility of explaining magnetic storms which come without obvious simultaneous increases in solar activity. The energetic solar particles are stored in the outer radiation belt, and released by some mechanism not directly related to an event on the sun. Notice that, as usual, in science the answer, may be only a partial answer, to one problem, immediately raises another. But it is a step forward in the understanding of solar-terrestrial interrelationships.

I could go on giving tid-bits of new knowledge gained from recent rocket, satellite and probe experiments; but these are such crude and simple experiments compared with the ones to come that I think it would be better to shift to another tack.

One of the important recent developments in space research has been the marshalling of scientific talent in this country and abroad to tackle the more sophisticated second and third generation experiments to come in the next few years or decade. The tasks are too big and varied to be carried out by lone scientists or a few laboratories. The approach to space science must be on a planetary scale, for our dimensions are making national boundary lines truly of insignificant importance.

In order better to understand current trends in the organization of

space research, it is useful to invoke the historical perspective furnished by the IGY. The program of the IGY represented an effort of unparalleled scope in the history of organized science. Because its scope included many "sciences" and encompassed the planet, the International Council of Scientific Unions was asked to sponsor the coordination of the program. As you know, ICSU is essentially a non-governmental organization comprised on the one hand of 13 scientific unions devoted to the various fields of science and on the other hand of 45 "adhering bodies," usually the national academies or similar institutions in the various countries. For the IGY, ICSU set up a special committee, known as CSAGI after the initials of its French name, *Comite Special de l'Annee Geophysique Internationale*. CSAGI then called upon the adhering bodies to establish national committees which could serve to effect international cooperation in the IGY and to plan and direct their own scientific contributions. In this country, the National Academy of Sciences set up the United States National Committee for the International Geophysical Year, which assumed responsibility for the stimulation of interest and support for the IGY, the planning, the initiation of projects, and over-all coordination. The programs themselves were carried out by the scores of U. S. laboratories active in the IGY fields of science. The same pattern was followed in other countries.

The IGY succeeded brilliantly in marshalling international cooperation. Accordingly, although the observational and experimental phase of the IGY ended on December 31, 1958, it has already proved to be a firm foundation for continuing international collaboration in geophysics. Nowhere is this more apparent than in the field of space research.

As the IGY drew to a close, there could be no doubt that the interests of human progress and the national welfare demanded that a long-term program of space exploration be pursued by the United States with utmost energy. In keeping with the responsibility of the National Academy of Sciences as advisor to the government on scientific problems, the establishment of the Space Science Board was announced on August 2, 1958, by Dr. Detlev W. Bronk, President of the Academy. The purpose of this Board is, in the broadest terms, to ensure the constructive support of the scientific community for the realization of a sound and imaginative program of scientific research in space. On the domestic scene, its functions in this regard are to be the focus of the interests and advisory responsibilities of the Academy in space science; to draw the attention of scientists to the problems and opportunities in space research; and to provide advice as they may require. In international scientific affairs, the Board acts as the instrument for collaboration with the International Council of

Scientific Unions and serves thereby to promote the cooperation of American scientists with internationally coordinated programs of space research.

Responding to the challenge of space in the pattern which proved so successful during the IGY, in October 1958 ICSU adopted a resolution establishing the international Committee on Space Research (COSPAR). The function of this Committee is to assist in the advancement on an international scale of fundamental research carried out with the use of rockets or rocket-propelled vehicles. The membership of COSPAR includes representatives of the appropriate international scientific unions, the scientific institutions of those countries having satellite or other space research programs, and of other countries having special interests in space science. Our Space Science Board, in its international role, represents the interests of the U. S. National Academy and the American scientific community on COSPAR.

Three working groups have been established by COSPAR to study, respectively, (i) tracking and telemetry, (ii) proposals for scientific experiments and research programs, and (iii) arrangements and methods for exchange and publication of data. In addition, the functions of the ICSU Committee on Contamination by Extra-terrestrial Exploration (CETEX), which had been established before COSPAR came into existence, have now been transferred to COSPAR.

In essence, COSPAR provides a means for continuation of the cooperative IGY programs of rocket and satellite research. Please note that in COSPAR as in the IGY, we have scientists talking with scientists, not governments with governments. While no mechanism works perfectly, I want to cite the evident success of the IGY as a real example of the potential of this mechanism for coordinating major international scientific programs. One may confidently expect that COSPAR will be able to broaden and intensify the fruitful scientific relations which characterized the IGY programs.

I should not want to give you the impression that the Academy's Space Science Board is an operating agency. It was noted at the time of its establishment that, in accordance with the Academy's tradition, the Space Science Board would function solely in an advisory capacity. The establishment of the National Aeronautics and Space Administration, on October 1, 1958, provided the means for the continued development and vigorous prosecution of a civilian program of space exploration. Responsibility for military and defense aspects of the conquest of space are, of course, in the capable hands of the Advanced Research Projects Agency of the Department of Defense. Support for work in space research is also provided by the National

Science Foundation. Thus, responsibility for the operational aspects of the conduct and support of space research has been vested by law in these government agencies.

The advisory functions of the Academy's Space Science Board are of a dual nature—not only to serve the needs of the government but also of the scientific community by giving audience to and advice on the problems and suggestions of individual scientists and research institutions. At the time of the Board's formation, the National Aeronautics and Space Administration had not yet come formally into existence. In order to assist in the formulation of the beginnings of a sound research program, the Board took over the initiative of soliciting proposals and suggestions for research in space, a task which had already been appreciated and begun by the Satellite Panel of the Academy's IGY Committee. The widespread response to these requests provided the basis for a series of recommendations for a beginning research program. It is gratifying to note that these recommendations have, essentially in their entirety, been incorporated into the research program of NASA.

In view of the success of these initial endeavors and NASA's rapidly developing strength and competence, the Space Science Board now feels free to devote its efforts more particularly to the consideration of the longer term problems in space research, to study the support which space research may require in the future from other related branches of fundamental scientific inquiry, and to examine the problems of individual research workers in universities and elsewhere.

I believe a word here about the space science program of NASA, which was developed as I said with the advice of the Space Science Board, would be of interest. I cannot, of course, speak for NASA, but their program has been announced in some detail. This program has been developed on as broad a basis as possible. Although much of the program planning is still in its preliminary stages, published information indicates that in each of the next two years it is hoped to launch between 75 and 100 sounding rockets, and on the order of one or two satellites or space probes every two months.

In the *rocket sounding program*; emphasis will be placed upon experiments relating to atmospheric structure, electric and magnetic fields, astronomy, energetic particles, and the ionosphere.

The *satellite program* will emphasize atmospheres, ionospheres, astronomy, energetic particles, electric and magnetic fields, and gravitation. A relativity experiment is of special interest.

*Space probes* will investigate energetic particles, fields, and ionospheres.

*Ground support programs*; such as, ionospheric sounding, solar

activity, etc., give associated data essential to the interpretation of many experiments in space vehicles. Other experimental work on the ground includes tracking, orbit computation and telemetry recording.

What can we learn from such a program? A summary of the knowledge to be gained in one specific area is revealing. Urey notes that a satellite of the moon could be instrumented to investigate whether the surface of the moon is granitic, basaltic or meteoritic in composition and whether the composition of the maria and land areas are different. High resolution television of the back and front areas of the moon would add to our knowledge of the relationships of maria and land areas. The mass and mass distribution can be secured from orbital constants of a satellite. The density and composition of the atmosphere can be secured by use of suitable mass spectrometers. Exploration of the lunar magnetic field with high-sensitivity magnetometers and tests for trapped low-energy particles (a lunar Van Allen effect) could be made. Of course, even more extensive information could be obtained from landings on the surface which will come eventually,—first hard, then soft, and finally manned.

I have said earlier that the age of space gives promise of a limitless frontier to the human mind and spirit. The many problems of this new era also present a challenge which will need to be met and conquered. Not the least of these is the challenge to engineers. Ever larger and more efficient space vehicles must be built. New propulsion systems producing greater thrust and improved engine performance must be developed. New standards of vehicular reliability must be achieved. More and more sophisticated instruments will be needed. Undoubtedly, tomorrow will bring new problems unknown to the engineer of today.

From all that we know of the history of science and engineering, solution of these problems will be difficult, but not impossible. We cannot doubt that they will be solved, for the world of tomorrow holds out the ultimate promise of knowledge and understanding beyond our fondest dreams. It is small wonder then that the Space Age, arising from the IGY, offers such a thrilling challenge as a companion to its hard problems.

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#### IONS PRODUCED EASILY

An easier, cheaper way to produce negative ions was discovered here when aluminum foil was exposed to ultraviolet light from germicidal lamps. The new system is many times more efficient than the old in producing negative ions. In nature, negative ions are produced by rain, thunderstorms, cosmic rays, radioactivity, ocean spray and ultraviolet radiation from the sun. The laboratory method knocks electrons loose from the aluminum foil under ultraviolet light radiation. The electrons combine with air molecules to become negative ions.



## Physical Science in a Modern Science Curriculum

Roy D. Meiller

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Township High School District #214, which includes Arlington High School, Arlington Heights, Illinois, and Prospect High School, Mount Prospect, Illinois, offers a complete and unique science program. It is complete in that the courses offered include biology, botany, zoology, physical science, chemistry, physics, and earth science. It is unique in that it is one of the few high school systems that does not offer general science, but instead, offers biology at the freshman level. Approximately ten years ago general science was dropped from the list of courses and biology was instituted at the ninth grade level. Physical science was then introduced at the tenth grade level to serve two purposes: first, to act as a terminal course in science for those students who would go no further in the science field; and second, to act as a good stepping stone for those students planning to continue in the sciences. The physical science course is not required and, therefore, there are some students who enter physics and chemistry classes who have not had this course. It has been found through experience that these students find chemistry and physics more difficult and that classes, necessarily, moved more slowly. Therefore, physical science is recommended for all students, especially for those planning to continue in any of the areas of science. Although the course was designed primarily for the tenth grade, juniors and seniors may enroll.

The staff of the Science Department of District #214 believes that education, especially in science, consists of developing or enhancing innate qualities or potentials by enlightening experiences. It is believed that this philosophy can be achieved by striving for several objectives. An awareness of the impact of science on our present day living, and an understanding of one's place in an environment for more effective living in a complex society must be developed if an adequate curriculum is to be achieved. A working vocabulary and reading ability of science literature as a means of communication with others must be developed if the student is to be trained to understand problems and acquire an ability in problem solving. The student must be provided with an opportunity for using the basic instruments and for developing skills in pursuing the facts of science. A science program can never be adequate unless the student is trained to achieve satisfaction and to relieve personal tension by developing scientific interests and an appreciation of science. The introduction of the physical science course makes it possible for

science education in District #214 to meet successfully the above philosophy and objectives.

Physical science as taught in this school system is a basic course in the foundations of the physical sciences and the application of these concepts to the experiences of the students involved. The course includes two hours of laboratory work per week in addition to three hours of discussion per week. The laboratory work consists of standard chemistry and physics experiments conducted in chemistry and physics laboratories. In addition, numerous incidental demonstrations are performed whenever appropriate.

Approximately sixteen weeks are devoted to physics units with the time being portioned out to units in forces, forces and motion, work-energy-power, engines, heat, sound, light, and electricity. Approximately sixteen weeks are devoted to chemistry units with time being portioned out to units in structure and behavior of matter, oxygen, hydrogen, water, atomic and molecular theory, formula writing, and reactions and equation writing. Two weeks are devoted to a study of astronomy and two weeks are used studying earth science.

The manner in which the various units are covered is flexible and is determined by the ability, interest, and needs of the students. Many of the topics in the above units lend themselves to rigorous mathematical derivation and application. The underlying philosophy of this course, however, is to emphasize the qualitative aspects of the subject matter content. The use of measurement, ratios, proportions, and simple equations is encouraged where it will enhance the meaning and practical application of a particular concept. The possibilities for correlation of different topics is always taken into consideration.

An assigned text is used and is supplemented with various reference books. Although the number of good physical science textbooks was inadequate a few years ago, there are now several good texts on the market. Even though a physical science text as such might not be available there are certainly excellent chemistry and physics texts available at various levels of difficulty.

In order to account for individual differences the practice of ability grouping has been introduced in physical science. Students may be moved from one group to another as is warranted by the student's achievement. The practice of grouping allows any student to work to the maximum of his ability and still to work with other students with similar abilities and achievements.

It has been found through experience that this course is not as difficult to teach as the general science course. In order to teach general science adequately, the instructor needs a background in the



physical sciences, astronomy, earth science, and biology. It is difficult to find an instructor with such a wide background. By not including biology, the physical science instructor has become more specialized and can do a better job if confronted with only the physical sciences. By narrowing the background requirements, the task of locating new instructors becomes easier. It has been found that there are many instructors with a background of both physics and chemistry, even though they may be more specialized in one of the subjects. People interested in the teaching of science realize the close relationship between chemistry, physics, and mathematics, and thus it is common to find instructors who have a background in all three fields.

The physical science course, which can be either terminal or preparatory in nature, certainly has advantages over special terminal courses of chemistry or physics. It has to be realized that students are preparing for a very complex future. With the use of a course such as physical science it is possible to give the student a greater background in both chemistry and physics. In fact, in District #214 this course has paved the road to the teaching of college level courses in both chemistry and physics.

With the adequate preparation which this course can give, college level courses in high school are possible. There seems to be little doubt of the value to the students. It means the possibility of participating in advanced placement examinations with the possibility of advanced standing in college work, or perhaps credit granted by the college.

The students are extremely enthusiastic about physical science. Although to some it is strictly a terminal course, to others it creates an interest in science which otherwise would have remained dormant. It has caused an increase in the percentage of the student body who participate in the science curriculum. In 1954, 58.8% of the student body at Arlington High School was enrolled in at least one science course. In 1957 a peak was reached in which 65.1% of the student body was enrolled in the science curriculum. Records at Arlington High School since 1957 are not conclusive because the school district has been in the process of being divided into two high schools. In a school with an enrollment of more than 2,000 students this is an increase of a substantial number of students.

We do not conclude that we have the ultimate science program. Through experience we are finding the science program in District #214 meets the present day needs of the students. It is the hope that modifications can be made to our program as soon as the needs of the student are not being completely satisfied.

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## Developments in Teaching Physics to Engineers at Yale\*

W. W. Watson

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Almost four years have elapsed since the publication of the report of the American Institute of Physics Committee on the Role of Physics in Engineering Education. I was a member of that committee and I strongly endorse its conclusions. In case you did not see this report at that time, I can assure you that it is not particularly "dated." It still makes good reading, and Dr. Elmer Hutchisson, the chairman of that committee and now director of the American Institute of Physics, tells me that a reprinting will soon be available.

At Yale we have made no revolutionary, sweeping changes in our physics course for engineering students. Rather, this is just a discussion of progress in our attempt to follow the recommendations of the AIP committee. We are limited to the fairly common arrangement of the 10 semester-hour course in the sophomore year. This time is divided into 3 hours of recitation in sections of 24 students each, one hour of demonstration lecture in sections of about 100 students each, and a 2-hour laboratory for the very same students as in one of the recitations. The same instructor is in charge of the recitation and the laboratory, with a graduate student aiding him both in the laboratory and in reading the problems for the recitation.

If the physics course for engineers must be confined to the sophomore year, then we should make the best of it and take full advantage of the freshman-year program. First and foremost, this means that calculus methods can be used from the start, for freshman mathematics for engineers should always have introduced the students to the ordinary operations of both the differential and integral calculus. And since our freshman chemistry course, required of all engineering students, is now being taught from the point of view of atomic structure, we can assume that they already have been introduced to the kinetic theory of gases, electron shells, ions, molecular sizes, etc. Also, since engineering mechanics has been taken for one term of the freshman year and is being studied further concurrently with the physics, we are shortening the time devoted to mechanics. Statics and elasticity, for example, we have now nearly eliminated. Thermodynamics and considerable portions of the section on heat are now given minimal attention, since we realize that all engineering students will study these topics in other courses.

\* A paper presented at the joint conference of the Mathematics and Physics Divisions, 1959 Annual Meeting of the American Society for Engineering Education.

With the purpose of bringing about better integration of physics and engineering courses, we held one highly successful joint meeting of the physics and engineering faculties, and we have followed this up with separate discussions with the individuals most concerned; in electrical engineering, applied mechanics, and with the Associate Dean of the Yale School of Engineering. We physicists were pleased to learn that the engineers wanted us to emphasize ideas and basic principles, with particular stress on the modern topics that engage the attention of researchers in physics these days. There was also unanimous agreement that these modern topics should be distributed all through the physics course, not compressed into a couple of chapters at the end.

As a result of these discussions, we have also segregated the electrical engineers into exclusive recitation-laboratory sections. These students attend the same demonstration lectures, follow in general the same assignments, and take the same term examinations as all the other students, but in the recitation and laboratory sessions their instructors can take full cognizance of the material in a parallel course in the EE department. In particular, some of the DC-AC circuit topics are eliminated and replaced by fuller discussion of modern atomic, nuclear and solid-state experiments. Also, for these EE students we require more analysis. For example, they go further into vibrating systems such as a ball in a cylindrical track, a rocking segment of a solid circular cylinder or cylindrical arc, a rectangular bar on an anvil, or a U-tube partly filled with mercury.

This presentation of more challenging and difficult material to the EE students, within the framework of the general engineering physics course, is working well. I recommend choosing the instructors for these special divisions with care, for they should be men who believe in this co-operation with the staff of the EE department, are willing to give close attention to making the laboratory experiments interesting and challenging, and are themselves good experimental researchers. It is better to treat the EE majors in this way rather than to organize a separate physics course just for them because it is clear to these students then that they are still held responsible for all of the material in the course and that all of it is worthwhile. Also, administratively it is not desirable to proliferate the number of general physics courses. After all, "physics is physics," and this strong trend towards emphasizing modern 20th century physics and sloughing off the minutiae and the well-known engineering applications of classical physics is the proper treatment for all categories of students.

I confess, however, that at Yale we have now a freshman-sophomore two-year course for students who come to college thinking that they may be physics majors, and this course is becoming increasingly

popular. Our potential physics majors are electing the sophomore half of this course two to one over the standard engineering physics course, and there is evidence that physics is eating into the registration of the Engineering School. This is occurring without any proselytizing on our part, and it may well be the post-Sputnik trend in many of the universities of this and other countries. Perhaps it is time for all engineering faculties and physics departments to join in offering an engineering-physics program, both at the undergraduate and the graduate level. We have no such program at Yale, but I for one would welcome discussion about starting one if our engineers proposed it. I firmly believe that two courses that should be required in an engineering-physics major beyond the sophomore year should be one covering atomic, nuclear and solid-state physics, and the other a course in electronics, both lectures and laboratory.

Modernizing and extending the mechanics portion of the physics laboratory sessions is not easy. Some of the ideas we are working on are: (1) using a set of electronic decade scalars together with radioactive sources in the first experiment to teach the theory of errors; (2) one or more basic gyroscope experiments with a large, well-built, electrically-driven gyro that has recently come on the market; (3) the physical pendulum experiments already mentioned on which any amount of analysis to fit the student's abilities can be required. As to the first of these, students like to use modern equipments, and even though they do not understand the electronic circuitry, they are placed in the correct state of mind to be willing to learn something about the treatment of a random set of data. The gyroscope is not new, of course, but it fascinates the students, and if they can become thoroughly familiar with axial vectors and the basic principle of the gyro, what they learn will come them in good stead in these days when inertial navigation, the gyro pilot and the precision of the electron's orbit must be well understood.

Another topic in mechanics which is guaranteed to interest the students these days is projectile flight and satellite motion. They seem to enjoy the ballistic pendulum experiment, using a modern B-B gun. The discussion of rockets presents a fine opportunity to discuss the conservation laws. Satellite motion is, of course, a "must" these days, and we spend some time on the theory of relativity which comes in logically at this point. Our discussion of the inverse square law of attraction ends with the simple Bohr theory of circular orbits for the hydrogen atom. Remember that our sophomore students have already had some introduction to electron orbits in their modernized freshman chemistry, and we are trying deliberately these days to introduce modern concepts into every part of the course.

Even though they already know some kinetic theory, we spend

about a week on this topic. This is largely 19th century physics, of course, but here for the first time the student is introduced to the statistical treatment of large numbers of events. Think of the recent developments in vacuum engineering, and the diffusion of neutrons in a moderator and of electrons and holes in a semiconductor! Such modern applications, with more to come, indicate that here is one subject in classical physics that must not be skipped.

In electricity we use the electron as the unit of electric charge from the start. The question of units is a thorny one, of course, but we are tending more and more to use MKS units throughout the course. I notice that there is no agreement among engineers about units, however, for the EE department seems to use MKS units exclusively, whereas the engineering mechanics staff uses english units. You engineers ought to get together on the subject of units!

There are many opportunities to introduce modern developments in the electricity section of the course. We discuss particle accelerators such as the cyclotron, the Van de Graaff machine, the Cosmotron, linacs, and the Cockcroft-Walton voltage multiplier at appropriate times. Students are fascinated by these modern machines, and they learn all the first principles of dynamics and electrodynamics in this study. In discussing the effect of magnetic fields on moving charges, the mass spectrometer also comes up as a natural example, and this discussion leads into isotopes. That is, quite a lot of nuclear physics can be injected naturally and profitably into this electricity section, and it is our experience that the students like it.

Basic electronics, the principles of rectification and amplification, we emphasize both in class work and in the laboratory. Every student now also does an experiment with the transistor. The applications of electronics are so numerous these days that we would be derelict in our duty if we did not drive away at the first principles that are involved in these devices.

Since there is a fairly good, reasonably-priced, cathode-ray oscilloscope kit on the market, no general-physics laboratory should be without a full set of them. We use these scopes in several of the electricity experiments. Students like to use them, of course, for they realize that this is the classroom version of research scopes that they see in every radio and physics research laboratory. Our experience with these scopes has been good.

Another example of classical material we have had to scuttle in order to make room for more of the modern atomic and nuclear developments is geometrical optics. One hopes that the introduction to the laws of mirrors and thin lenses in high-school science courses will suffice. The topics of polarization, interference and diffraction in physical optics must be retained, however, for the understanding of

the first principle of these topics is basic for later work with many modern developments. We also have the students do experiments on interference fringes and diffraction gratings of several dispersive powers.

Finally, in our endeavor to amplify the treatment of modern physics, we are introducing some nuclear experiments, but this is not easy in the 2-hour laboratory period of the sophomore course. However, we now have a successful experiment on the half-life of indium, using a good low-priced scaler now available. We make the indium foils radioactive at the start of the laboratory period by exposing them to a plutonium-beryllium source at the center of a large bucket of paraffin. We are now also using a diffusion chamber (there is a low-priced, useable one now commercially available) for a semi-quantitative experiment measuring alpha-particle ranges.

In summary, then, I can report that all of our moves in the direction of the 1955 recommendations of the AIP Committee have been successful. I can make no claims to outstanding performance, for I realize that our communication between engineers and physicists at Yale could be improved, that our teaching of physics to engineering students could be better if we were allowed more time than that of the 10 semester-hour course, and that it might also be better if all of us physicists with this teaching assignment spent more time on it and less on research.

On this last point, however, I maintain that especially in this preprofessional physics course, producing scholars should be the most stimulating teachers. They should be able to more than make up for any lack of smoothness in their performance before the class by their enthusiasm for research in physics and their obvious knowledge of many current developments. My own first contact with physics was in a laboratory where research was the order of the day, and I am certain that I was so stimulated by that atmosphere that it definitely influenced my entire career.

I also feel that to get more of our ablest students in science and engineering to go into college and university teaching, research opportunities must be available to them along with their teaching. These best students are going to shy away from teaching if they have teaching loads of 15 hours per week, with no chance for engaging in productive scholarship. But almost any scholar is willing to spend some of this time teaching the fundamentals of his subject, and he is probably a better professional physicist or engineer for having engaged in this training of newcomers in his field.

In preparation of this talk, I have reread that 1955 Report on the Role of Physics in Engineering Education. It is not at all "dated." In my opinion, we should continue to use it as a guide.



## The Education of Secondary and Collegiate Teachers of Mathematics

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A major item in any discussion of curricular revision in mathematics is the availability of teachers able and willing to offer instruction in the subject matter, both as to outline and in the spirit of modern mathematics. Decisions for or against teaching as a career frequently occur at the beginning of the college years and are perhaps elicited by a strong or weak teacher and/or by adequate or inadequate subject matter.

There is so much in mathematics to expose in so little time that any program must emphasize comprehensive long-range ideas and the choice of subject matter must be determined not merely from a reputation for traditional usefulness but also by current interest. A host of topics suggest themselves for inclusion in secondary teaching preparation, for example: set theory, probability and statistics, abstract algebra, symbolic logic, etc. However, numerous traditional and standard topics must also be kept in the curriculum. It is not the content but rather the presentation which will be changed, as for instance algebra which should emphasize logical structure instead of mainly skills.

Regardless of the preparation of the present secondary school mathematics teachers, the undergraduate program for future teacher presentation should be modified and as soon as possible to provide sound contemporary mathematics and to produce teachers prepared to dispense the new mathematical patterns. The teachers' most urgent need in order to propagate the new curriculum is not methodology (although a certain amount is necessary) but subject matter.

The Commission on Mathematics of the College Entrance Board in a pamphlet "The education of secondary school mathematics teachers" (Sept. 1957) states that a very adequate teacher education program can be developed around a major of 24 semester hours beyond the calculus. This is an excellent suggestion, long overdue, and certainly the optimum in challenge since at present many institutions accept as a teaching major 9 to 12 hours of mathematics beyond the calculus. The recommended major of 24 hours would imply two mathematics courses per semester for the junior and senior years and this need not be an extreme burden.

The writer would like to add as a comment that all the courses listed in the Commission's pamphlet (page 15) and contained within the suggested work below could and should be part of the preparation of a secondary mathematics teacher. Any of the work not included in

the major might later be taken in summer sessions or evening classes.

Considering the mathematics content of the courses for prospective teachers, the following could be suggested. I. Principles of mathematics: concepts regarding logic, number systems, groups, rings, fields, sets, theory of equations, algebraic functions, with the traditional necessary topics, and analytic geometry with an introduction to exponential and logarithmic functions, limits and calculus. II. A thorough course in calculus along with solid analytic geometry through quadric surfaces and some theory of determinants and matrices. III. Advanced calculus, differential equations, and a first course in each of probability and statistics and plane statics and dynamics. IV. Foundations of mathematics, projective geometry, history of mathematics, theory of numbers.

Supporting courses might include two years of English, one year of French or German or Russian stressing reading ability, one year of physics with a semester's work in each of chemistry and biology followed by another full year's work in one of these same three, a year of history, along with courses in psychology, philosophy, economics, political science and education to make up a full schedule of some five courses per semester.

Some thought might also with profit be given to the master's degree program which might follow a major such as outlined above. Colleges are finding it difficult to find sufficient Ph.D.'s to staff the mathematics departments. A much strengthened master's degree might be of considerable assistance and would provide excellent teacher performance at the college level.

This writer would venture the opinion that the master's degree program should encompass two years beyond the recommended major of 24 hours and should embrace in substance almost all of the academic preparation at present required of the doctoral candidate but minus the thesis. The doctor's degree could later be conferred on those individuals who further specialize and who by way of thesis present an original and significant contribution to knowledge.

The master's degree program could consist of two full years of function theory including integration theory, one year of modern algebra including Galois theory and one year of topology and advanced analysis including the calculus of variations. These would comprise half the work and could be required.

The other half would consist of electives and might include courses in analytic projective and differential geometry, courses in advanced mechanics, courses in actuarial mathematics, the differential equations of mathematical physics, the mathematical theory of electricity and magnetism, the mathematics of relativity, etc.

The electives would of course depend upon student demand and current staff.



## Organization—In Preparing a Technical Paper\*

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Our panel topic, "The Communications Problems . . ." reminds me that when I was asked to serve as a member of the panel this afternoon, it was stated that this was a joint meeting with the English Division of ASEE. Without knowing too much about the organization of the Society, it was assumed that this was an international meeting, with the English (British) Section participating—another, or different problem in communications?

In order to be better informed for the making of a few remarks on preparing an engineering or scientific paper, I thought that I had better peruse some of the published books on the subject. And in looking them up, I found that the books on technical writing are found on the same shelves as books on engineering as a career, etc. The proximity might indicate an inseparable connection between the two—and perhaps there is. Certainly for the engineer, interested in and working in research and development, the publication of and presentation of papers in his field is a usual collateral activity. However, I know of several or even many successful engineers, who have risen to top levels in industrial organizations—who have never published a paper. Not that they aren't effective writers—of reports, of memoranda, or letters—in many or most instances. To be articulate, in speech or writing, is certainly a considerable attribute in "communicating"—a somewhat labored vogue-word, that means primarily to impart or bestow (information, etc.) and secondarily, to share or exchange information, etc. However, it perhaps should not be overlooked that many people enjoy the act of writing or giving a talk, and others do not—perhaps because of their inabilities along these lines. Einstein once said, with great humility and truth, I think, that his motivation in his scientific work was not really to advance the scientific knowledge of the world, etc., etc.—but was that he enjoyed the subject, and the challenge of the unsolved and unknown absorbed him greatly.

Preparing a paper usually means writing a paper and presenting it before some technical society. Both aspects have their own spheres of importance, because while large segments of the technical public may hear the presentation, probably larger segments will only read the paper. This last phrase is intriguing, because of its double mean-

\* Presented at The Joint Meeting of The English Division, American Society of Engineering Education and The Pittsburgh Chapter, Society of Technical Writers and Editors During the 67th Annual Meeting of ASEE Pittsburgh, Pa., June 17, 1959.

ing and usage. If I were to ask an academic friend of mine what he is doing at the Chicago meeting of the X.Y.Z. Society, he might say, "I am reading a paper"—although he might say, "I am making a speech," or if awed by the latter phraseology, he might say, "I am giving a talk." Unfortunately, many engineers do not "read their paper" in the English sense or usage of the phrase, meaning "discoursing on a topic," but in the American literal sense, of actually reading aloud from their manuscript. While this is not a part of my topic this afternoon, I cannot let the matter pass without saying that one of the main weaknesses of engineers' presentations of papers is the reading of their manuscript to their audiences, rather than giving a talk on the subject, following the context of the manuscript, which may be read later at leisure.

The writing of a paper is usually or perhaps always a chore, for engineers primarily concerned with carrying out an investigation, or designing a machine, or the like. The requirements that he report his knowledge in a coherent and easily understood and appreciated form may involve more effort than doing the actual work. It has sometimes been said that a good expositor is born and not made, or that one cannot be taught to write by following a set of rules. Perhaps it is true, but the writer feels that the subject has not always been properly taught in our engineering schools and universities. In the writer's view, engineering schools should not give a first year course in "English for Engineers," but should require engineers to take the same course in English that all other university students take, or if the engineering or technical school is an independent entity, the English course should be of the same high standards. The very title implies curtailed or restricted English, perhaps of the handbook variety. And the course should be thorough and exacting, requiring the writing of one or more themes or compositions per week. "Too tough," one may say—or perhaps, all too many say—these days. The writer remembers his freshman English course at college as the toughest course of his entire four years, requiring the writing of a short story, a legal brief, etc., toward the end of the course, in addition to the weekly themes. I would comment, looking back now, that for engineers the brief might well be substituted for by a technical paper or two. In this regard, the continued practice of writing, critically reviewed by the instructors and professors, is without doubt the best training for engineers who are later to write technical papers. Good English usage, correct grammar and punctuation, etc. are of as great value to engineers writing technical articles as for lawyers or statesmen or authors writing legal tracts, or political documents, or novels.

A technical paper should be a calm, orderly, and objective presentation of a subject. The excessive usage of flowery or emotional lan-

guage will certainly detract from its value and interest. Any lack of orderliness in the development of the subject, such as is frequently used in newspaper reporting, may lead to confusion. The paper, by all means, must be readable. One of Oscar Wilde's famous witticisms is quite applicable to technical writing, when he said "The difference between literature and journalism is that journalism is unreadable, and literature is unread." While it is usually held that technical writing should employ short sentences, and simple direct statements, and the like, the writer cannot help but admire the longer, more involved, elegant sentences of Walter Lippmann, for instance, as well as the more direct, staccato style of Ernest Hemingway—even in a technical paper.

If one attempts to write a technical article without first preparing an outline, the result is almost certainly to be poor comprehension by the audience or the readers. Even if one were to use the rigid rules of the college laboratory report: (a) purpose, (b) scope, (c) discussion of results, (d) conclusion—the result would be better than no outline at all. In general, the subject should have an introduction, followed by the body of the paper, and ending in some form of conclusion.

The introduction serves the purpose of getting the readers acquainted with sufficient background on the subject being presented, to cause the points being made in the body of the paper to stand out in significant relief. And here I wish to emphasize an aspect of introduction that is usually slighted. The historical background of a subject is all too often only superficially mentioned, or not referred to at all. True, it requires a good deal of reading and library research, as well as a greater familiarity with the subject. But the history of a subject adds an interest that is usually neglected, both in our schools as well as in our papers. My plea would go even further—that all technical courses should devote a portion of their time to the historical side of the subject being taught, as I feel that it serves as an inspiration to the young students, and even to the mature engineers later on.

Once an outline has been established, the writer will find it desirable to adhere to the skeleton of the paper thus established. The body of the paper, of course, must develop the subject in a logical manner. To be good reading, however, the subject matter must flow smoothly from topic to topic and paragraph to paragraph—not proceed jerkily. As each new paragraph is begun, it should have a prefatory statement to introduce the subject matter to follow. Even if one were to write, "Let us now take up the matter of so-and-so," it would be better than no statement at all—however stereotyped and monotonous it may be. Transitional introductory sentences go far to blur the jerkiness of the formal outline, and smooth the flow of the presentation.

For instance, such a statement as "Although the rolling of steel is usually carried out on four-high mills, the cluster mill has its particular forte" is to be preferred to "Let us now turn to the cluster mill." The use of transitional sentences and sometimes paragraphs parallel somewhat the device of the photographer in touching up his negative or using special filters to give softer, smoother, blending lines—introducing some character and style to the picture that otherwise might be too stark and contrasting.

It may be elementary to many, but this writer would comment that the proper use of figures and graphs or curves (rather than tabulated data) is desirable, particularly in the presentation of mathematical developments of a particular topic or phenomenon, let us say, and in presenting the final results. It is still difficult for this writer, for instance, to read the papers of the old and famous French Academy, under the title of *Comptes Rendus*, where figures are usually omitted, and mathematical symbols found scattered randomly throughout the running text. Where a considerable amount of mathematics is involved in a paper, it is the writer's preference that it be relegated to an appendix, repeating however the key equations in the main body of the paper, as part of the text. In contrast, however—and possibly contrary to the rules of most technical societies—this writer prefers to list bibliographic references in footnotes on the pages as they occur rather than having them grouped at the end of the paper. The continual thumbing of the manuscript back and forth to follow the references, interferes considerably with easy reading of the paper. The books of Professor Stephen Timoshenko are a good example of what I mean in this regard.

Just as a technical paper must have a conclusion to bring into focus the main points or point of view that the writer wishes to make, so much my remarks. I cannot help but feel somewhat apologetic in talking to a group of teachers of English and specialists in technical writing who are certainly far more knowledgeable and sensitive to the subject of technical writing than I can be. My remarks are made primarily from the standpoint of one who has had a lot of technical paper writing to do, and of one who feels that stiffer courses in college English would help in a major way toward turning out engineers capable of writing high quality technical articles.

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#### COMET FOUND IN SOUTHERN SKY

A new comet, the first to be reported in 1960, has been discovered in the southern sky. The object has been named Comet Burnham after its discoverer, Robert Burnham of Prescott, Ariz., who found it while working at Lowell Observatory, Flagstaff, Ariz. Mr. Burnham also spotted the first comet of 1958.

Mr. Burnham first spotted the comet on Dec. 30, then photographed it again on Jan. 2. It is too faint to be seen directly, even with telescopic aid.

## Discovery of the Indirect Method of Proof in Geometry\*

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Few will argue with the premise that one of the major objectives of 10th year geometry in our high schools is to acquaint our students with the nature of proof and to establish its importance in their minds. To this end we have failed if the students do not obtain a clear understanding of the nature of the indirect proof.

The importance of the indirect proof need scarcely be established for it is generally known that there are several theorems in mathematics which can be proved only by the indirect method. Its importance in life situations has been emphasized by those who claim that approximately 50% of our reasoning is indirect reasoning.

With the importance and need of the indirect proof established, our attention then turns to the question of how this topic can skillfully be presented to a class so that they can understand its use and appreciate its beauty as a logical form. I assert that by allowing or helping them to discover it themselves and by encouraging them to use it often this can be accomplished.

My purpose, therefore, in this paper is not to discuss the logic behind the indirect proof but rather to discuss a method of introducing this topic to a 10th grade geometry class.

It is necessary to establish first the difference between a direct and an indirect proof. Since our school is quite basketball-minded, I use a simple example from that sport to illustrate the difference.

We assume a basketball game is in progress with a score of 27-21 in Valpo's favor when one of the spectators goes to the cafeteria for a coke. Upon his return, the score is 28-21. From this information we determine how the point was scored during the visit to the cafeteria.

The indirect reasoning is normally apparent first with the argument that it "has to be" a free throw. With some encouragement, it can be established that this phrase, "has to be," can be used because we know the two possibilities of scoring in basketball and because the suggestion of a field goal contradicts the score.

The direct method of determining what happened is quickly suggested with such comments as: "Ask a friend. Check with the scorekeeper. Consult the referee." A discussion of the two types of reasoning points out the differences between the two; and other examples from sports, crime, car breakdowns, test scores, etc. establish the point more firmly.

\* Paper presented at the Annual CASMT Convention, Chicago, Illinois, November 26-28, 1959.

As an assignment for the day, I give more problems similar to what were discussed in class and several so-called logic problems which can be solved only with indirect reasoning.

One example of these problems can be found in Gamow and Stern's book, *Puzzle Math*, and reads as follows:

As is well known to travelers in Europe, the passengers on British trains habitually keep the car windows open to get fresh air, and along with the fresh air they often get a good deal of smoke from the locomotives which, according to British custom, use coal as fuel. Thus it is easy to imagine that some smoke blew through an open window into a train compartment occupied by three well-bred Britons who were sitting stiffly on their seats minding their own business. As the result of that accident, the faces of all three passengers were smeared in spots by black soot, which presented an amusing contrast to their impeccable clothes and snobbish bearing. One of them, a lady called Miss Atkinson, raised her eyes from the book she was reading, and in spite of her perfect upbringing could not help chuckling at the sight that met her eyes. The two men were chuckling also.

But—and this may be characteristic of British nature—each of the three passengers assumed that his or her face was clean, and that the two others were chuckling at the sight of each other's faces. (Good manners prohibited them from looking directly at the object of interest, so that it was impossible to see who was chuckling at what.)

The situation lasted for a few minutes. Then Miss Atkinson, who was better educated than the other two, being a school teacher, suddenly realized that not only were the faces of the other two passengers smeared with soot but also that her own face must be. She took out a handkerchief and rubbed her face thoroughly from ear to ear and from forehead to chin. An inspection of the handkerchief proved that her conclusion was correct, but how did she come to that conclusion?<sup>1</sup>

This problem can be solved by having Miss Atkinson assume her face is clean and thereby reach the conclusion that the two men would have realized that their faces were dirty and would have cleaned them. This, of course, contradicts the circumstances.

Another favorite with the students deals with the familiar problem of three men seated in a row, each trying to determine whether a black or a white hat has been placed on his head.

I have found the students love this type of problem and the class discussion the following day is lively, to say the least. And it is in this discussion that important principles of indirect proof are brought out; i.e., the law of excluded middle, if one of two contradictory positions is proved to be false, the other must be true, and if the conclusion of a correct line of reasoning is false, the hypothesis is false.

We are then ready to graduate to a geometry problem. For the first problem, I elect to prove that the angles opposite two unequal sides of a triangle are unequal or the converse of this theorem. These two problems are non-trivial examples of the desired principles. Also, the students are well acquainted with the theorems which will yield the necessary contradictions. A more complicated example would

<sup>1</sup> George Gamow & Marvin Stern, *Puzzle Math*, The Viking Press, 1958, p. 77-79.



tend to distract from the proof itself. In this case, however, the desired result is quickly seen and the method of proof can be concentrated on.

No attempt is made to formalize as the problems are first discussed, for without formalization the desired solution is normally forthcoming from the class. It is only after the spirit of the proof has been accepted that we as a class develop a formal proof, using the principles we have accepted in previous discussion.

As a matter of semantics, I have found it better to talk about "what would happen if the conclusion of this problem were not true" rather than just saying, "Assume this is not true." It seems a minor point but it does help those students who cannot see the sense of assuming something to be true that they feel sure is wrong before the assumption is made.

Following the formalization of these two problems, other problems are introduced and discussed. It is then that we investigate some of the theorems to be proved by the indirect method. By this time, it is hoped that the students will have no feeling that a trick is being employed to prove something nor that we are avoiding the issue.

I ask some of the better students to prove again, this time using an indirect approach, some of the theorems we have already proved. And all are encouraged, during the analysis of a problem, to consider the question—what would happen if the conclusion were not true.

Concerning the results of this method of introducing the indirect proof in geometry, I make no claims. I do know that my students have no fear of using an indirect proof and many are happy when they can use it.

A portion of their enthusiasm is undoubtedly due to my enthusiasm; however, I cannot help but feel that the two additional days spent in introducing this topic are well spent. The fact that the students are willing to try an indirect proof on a problem when the text does not suggest such an approach leads me to believe the students have an understanding of this method of proof and a competency for its use.

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#### FIND SEVERAL MOSSES INHIBIT GROWTH OF BACTERIA AND FUNGI

Antibiotics from common mosses may result from some studies reported by a team of Arizona State University researchers.

Three mosses collected locally were found to inhibit the growth of *Micrococcus flavus*, *Streptococcus pyogenes*, *Candida albicans* and *Micrococcus rubens*, the scientists found.

Somewhat less than one-half pound—200 grams—of each kind of moss was mashed in a blender along with various solvents and the extracts salvaged.

Antibiotic activity varied, it was found. The extracts were not always effective against the same organisms. There is evidence that several antibiotic compounds may be involved since extracts of the same moss species by different solvents gave different results.

## Making Biological Field Trips a Reality\*

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Prior to my joining the faculty of a small southern university, I had taught in high school for four and one-half years. While in high school work, I was interested in getting my students to appreciate the local flora; thus it was that I devised a series of field trips which proved to be successful enough to warrant my telling you about them. It is my hope that you, profiting by my experiences, will enjoy, as I did, seeing the students' interest in local flora take a giant forward step.

Like most of you, I was faced with considerable limitations relative to biological field trips: first, our high school had class periods only forty minutes long; second, one field trip of the disruptive type—need for buses, prefects, official excuses for missed classes—might be tolerated, but several, never; third, my four sections in biology generally averaged thirty-two students each, giving a total of one hundred and twenty-eight for the trip; and fourth, no special facilities were available locally for such field work. With these limitations in mind, I planned a series of field trips specifically designed to circumvent these problems.

Our campus, had several acres of landscaping which included many trees. Investigation showed that the campus boasted twenty-two evergreen trees which were of six genera, and forty-four broad-leaved trees divided among nineteen genera. So, to implement my plan for field trips, I prepared separate dichotomous keys for the evergreen trees and for the broad-leaved trees. (See diagrams and keys.) The two keys were designed for our campus only; they were quite basic—including characters familiar to students of high school biology.

Initially, each student was equipped, during his regular class period, with a copy of the evergreen trees key; a map† of the campus which indicated the route of the first prospective field trip and which trees and in what order they were to be identified; and with a list of twenty-two numbered blanks corresponding to the trees' numbers. Thus each student could tell in advance where he would move when signaled, which trees were to be identified, and where to record his identifications.

With these materials at the disposal of the students, in each biology section on the day preceding the field trip we went through a practice

\* Paper presented at the Annual CASMT convention, Chicago Illinois. November 26–28, 1959.

† The campus maps (one for the evergreens and another for the broad-leaved trees) depicted the school buildings, streets, and sidewalks; interspersed were the numbered stations indicating the trees which were to be identified by the students. Arrows on the maps outlined starting positions, routes of the field trips, and terminal positions.



## DICHOTOMOUS KEY FOR THE CAMPUS BROADLEAFED TREES

1. All leaves project outward from stems. . . . . 2  
     Some leaves, called scale-like leaves, usually are pressed against the stems,  
     especially at the tips of stems. . . . . 5
2. Leaves are long (4"-6") slender needles which are in groups of two on stems.  
     The stems bear large (3") cones at right angles to the branches. . . . . Red Pine
3. Leaves are short ( $\frac{1}{2}$ "-1") and are needle-like. . . . . 3
3. Leaves have short stems attaching to the branches of the trees; the branches  
     are very rough where leaves have dropped off due to those parts of the leaf-  
     stems which remain on branches; many long (4"-6") slender cones are present,  
     especially at the tops of the trees. . . . . Spruce
4. Branches are not especially rough where leaves have fallen off. . . . . 4
4. The length of the underside of each leaf shows two white lines; small cones  
     ( $\frac{1}{2}$ "-1") are present on stems. . . . . Hemlock
- Leaves do not have white lines on undersides. No cones are present. Sometimes  
     (on female plants, i.e.) red berry-like structures enclosing seeds are seen on  
     stems; grows as a shrub. . . . . Yew
5. Stems are flattened; cones are small ( $\frac{1}{4}$ "); the leaves are spread in a fan-shape  
     . . . . . Arborvitae
- Stems are round; blue berry-like structures enclose the seeds; plant is a shrub  
     . . . . . Juniper

## IDENTIFICATION LIST

1.	9.	17.
2.	10.	18.
3.	11.	19.
4.	12.	20.
5.	13.	21.
6.	14.	22.
7.	15.	
8.	16.	

run. For this purpose the students were individually supplied with fresh-cut stems of Arborvitae and Yew as illustrative examples. This exercise permitted the student to accustom himself to the use of the dichotomous key, to orientate himself relative to the campus and its map, and to receive instructions about the forthcoming trip.

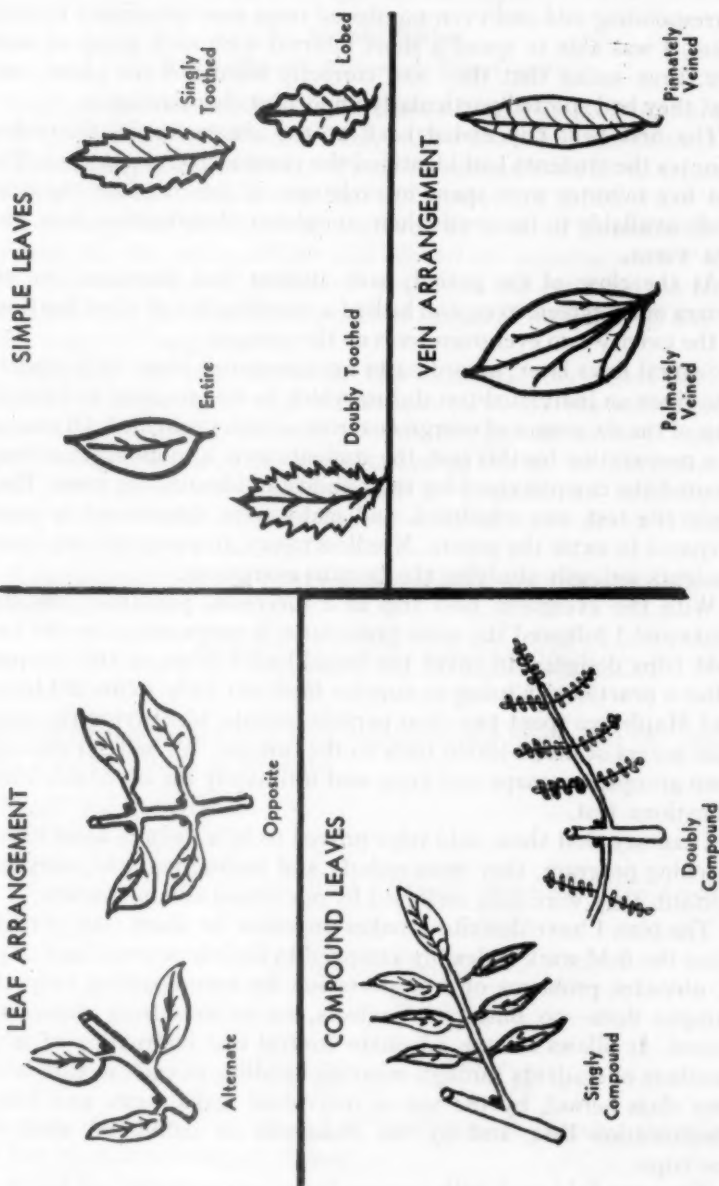
The next day, period by period, each section, armed with its keys, maps, and blank identification lists, moved from the classroom onto the grounds for a period of rather intensive work. Previously, at yesterday's practice run, the sections had been divided into halves—odds and evens—corresponding to the numbered trees on the map and on the blank identification lists. The odds were responsible for identifying only the odd-numbered trees and the evens only the

even-numbered trees. Whether odd or even, each student identified all six genera of evergreens, since the trees were numbered with this purpose in view. The advantage of the odd and even divisions was that it effectively split the sections into halves with corresponding reduction in confusion.

According to previous instruction, each group, as a unit, approached tree after tree, critically identifying the specimens using

#### DICHOTOMOUS KEY FOR THE CAMPUS BROADLEAFED TREES

1. Leaves are alternately arranged on the stems.....7  
    Leaves are oppositely arranged on the stems.....2
2. Leaves are compound.....3  
    Leaves are simple.....4
3. 3-5 leaflets on the stem; the fruit is 2-winged and hangs in clusters.....  
    Box Elder  
    7 leaflets are found on branches; fruit is like the Box Elder..... White Ash
4. Leaves lobes.....5  
    Leaves not lobed.....6
5. 3-5 lobes with sharp, tooth margins in leaves; veins often red..... Maple  
    Lobes are irregular; leaves may be "left glove," "right glove," or symmetrical  
    leaves have rough surfaces above and below..... Paper Mulberry
6. Fruit is a long (6"-12") bean-shaped pod with winged seeds inside. Catalpa  
    Fruit is a pod about 2" long; clusters of light brown buds project upward  
    from stem tips..... Pawlonia
7. Leaves are simple.....10  
    Leaves are compound.....8
8. Leaves are doubly compound; leaflets have a "midvein" which is not located  
    in the center of the leaflet..... Mimosa  
    Leaves are singly compound.....9
9. Stems have thorns; leaflets are 1" long on leaf stems about six inches long;  
    about 9 leaflets on a leaf stem..... Black Locust  
    Stems have no thorns; winged seeds hang in dense clusters on female trees;  
    11-31 leaflets each 3"-5" long are on leaf stems which are 1'-3' long. Ailanthus
10. All leaves are not of the same type; some are "left glove," some "right glove,"  
    and some symmetrical..... Red Mulberry  
    Leaves are not varied as they are above.....11
11. Leaves are palmately veined.....12  
    Leaves are pinnately veined.....15
12. Leaves are star-shaped; fruit is a spiny ball..... Sweet Gum  
    Leaves are not star-shaped; fruit not like the above.....13
13. Leaves are sharply toothed; seeds hang from leaf-like wings which are  
    2"-3" long..... Linden  
    No seeds attached to leaf-like wings.....14
14. Leaves are large (5"-7") and are lobed..... Sycamore  
    Leaves are small (2"), are not lobed, and are finely toothed..... Poplar
15. Leaves are broadly lobed; acorns present..... Oak  
    Leaves are not broadly lobed; no acorns present.....16
16. Leaves have entire margins; leaves are vividly red in fall..... Tupelo  
    Leaves have toothed margins.....17
17. Leaves have doubly-toothed margins..... Elm  
    Leaves have singly-toothed margins.....18
18. Leaves are light green and spear-shaped; are alternately arranged; fruit a  
    drupe..... Peach  
    Leaves darker green than above..... Wild Black Cherry



their dichotomous keys. Since the stations were arranged so that corresponding odd and even-numbered trees were proximate to each other, I was able to spend a short interval with each group at each tree, thus seeing that they had correctly identified the plant, and that they had spotted particularly important characteristics.

The first field trip ended back at the classroom; in thirty-five minutes the students had identified the campus evergreen trees. The last five minutes were spent in exchanges of information: the odds made available to the evens their completed identification lists, and *visa versa*.

At the close of the period, each student had identified the six genera of evergreen trees, and he had a complete list of identifications of the twenty-two evergreen trees on the campus.

Several days later, according to an announced plan, each student was given an individual test during which he was required to identify four of the six genera of evergreens from samples provided. Of course, as a preparation for this test, the students were a comic sight as they scoured the campus checking their ability to identify the trees. They knew the test was scheduled, and many were determined to come prepared to name the genera. Needless to say, it was gratifying to see students seriously studying the campus evergreens.

With the evergreen field trip as a successful precursor, the students and I followed the same procedures in preparation for the two field trips designed to cover the broad-leaved trees on the campus. After a practice run using as samples fresh-cut leafy stems of Linden and Maple, we spent two class periods outside, identifying the nineteen genera of broad-leaved trees on the campus. We had our odd and even groups, our maps and keys, and ultimately the inevitable identifications test.

I can say that these field trips proved to be a definite asset to my teaching program; they were orderly and instructive; and, very important, they were fully endorsed by our school administrators.

The plan I have described makes provision for short class periods, since the field work is flexibly arranged to include several class days. It obviates problems of transportation by encompassing only the campus flora—no buses, no prefects, no excuses from classes required. It allows for the adequate control and instruction of large numbers of students through separate handling of each section in its own class period, by the use of individual maps, keys, and blank identification lists, and by the insistence on individual work on the trips.

Try some field work with your students; you probably will find it as rewarding as I did. It takes only a little planning and a little flora.

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## A Triangle Theorem

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Suppose two cevians  $BP$  and  $CP$  meet sides  $CA$  and  $AB$  of triangle  $ABC$  at points  $E$  and  $F$  respectively. Allow a straight line through  $P$  to meet  $CA$  at  $N$  and  $AB$  at  $O$ . Under these conditions let us attempt to determine the value of  $AF/BF \cdot BO/AO + AE/CE \cdot CN/AN$ .

If we consider the segments involved as directed quantities it is evident that the ratios  $AF/BF$  and  $BO/AO$  are negative when  $F$  and  $O$  lie between  $A$  and  $B$ . If either  $F$  or  $O$  lie on  $AB$  extended the ratio or ratios must be considered positive. Similarly for points  $E$  and  $N$  with respect to segment  $CA$ .

Applying this convention to the accompanying figure let us consider triangle  $ABE$  with transversal  $CPF$ . By the well known theorem of Menelaus, we have  $AF/BF \cdot BP/EP \cdot EC/AC = 1$ . Again, considering triangle  $ABE$  and transversal  $NPO$ , we get  $AO/BO \cdot BP/EP \cdot EN/AN = 1$ . Since  $BP/EP$  is common to both equations, we may write  $AF/BF \cdot EC/AC = AO/BO \cdot EN/AN$  or  $AF/BF \cdot BO/AO = AC/EC \cdot EN/AN$ .

A further value for the right member of this equation may be determined as follows:

$$\frac{AC}{EC} \cdot \frac{EN}{AN} = \frac{(AE + EN + NC)EN}{EC \cdot AN} = \frac{AE \cdot EN + \overline{EN}^2 + NC \cdot EN}{EC \cdot AN}.$$

Adding to and subtracting from the preceding numerator the quantity  $AE \cdot NC$  allows us to write

$$\begin{aligned} \frac{AE \cdot EN + \overline{EN}^2 + NC \cdot EN}{EC \cdot AN} &= \frac{AE \cdot EN + \overline{EN}^2 + NC \cdot EN + AE \cdot NC - AE \cdot NC}{EC \cdot AN} \\ &= \frac{(EN + NC)(AE + EN) - AE \cdot NC}{EC \cdot AN} \\ &= \frac{EC \cdot AN - AE \cdot NC}{EC \cdot AN} = 1 - \frac{AE \cdot NC}{EC \cdot AN}. \end{aligned}$$

We may change  $NC$  to  $CN$  and  $EC$  to  $CE$  thereby leaving the value of the fraction unchanged. Hence,

$$1 - \frac{AE \cdot NC}{EC \cdot AN} = 1 - \frac{AE \cdot CN}{CE \cdot AN}.$$

Thus

$$\frac{AF}{BF} \cdot \frac{BO}{AO} = 1 - \frac{AE}{CE} \cdot \frac{CN}{AN} \quad \text{or} \quad \frac{AF}{BF} \cdot \frac{BO}{AO} + \frac{AE}{CE} \cdot \frac{CN}{AN} = 1$$

and we have a rather remarkable result. We may, however, reverse the direction of the segments in the denominators of the four fractions given above without changing the value of the right member of the equation thereby expressing our result in what seems to be a better form. Accordingly,  $AF/BF \cdot BO/AO + AE/CE \cdot CN/AN = 1$  becomes  $AF/FB \cdot BO/OA + AE/EC \cdot CN/NA = 1$ . When this change is made we must henceforth consider points  $F$  and  $O$  as giving positive ratios  $AF/FB$  and  $BO/OA$  when  $F$  and  $O$  lie between  $A$  and  $B$  or negative ratios when  $F$  and  $O$  lie on  $AB$  extended. Similarly for points  $E$  and  $N$  on  $CA$ .

It is not difficult to establish the converse result and we may now sum up our conclusions with the following

**Theorem.**  $BP$  and  $CP$  are two cevians of triangle  $ABC$  meeting sides  $CA$  and  $AB$  at points  $E$  and  $F$  respectively. A straight line through  $P$  meets  $CA$  at  $N$  and  $AB$  at  $O$ . Then  $AF/FB \cdot BO/OA + AE/EC \cdot CN/NA = 1$  and conversely, if this relation exists,  $NO$  will pass through  $P$ , the point of intersection of  $BE$  and  $CF$ .

When the theorem is stated in this form, let us emphasize again that if  $F$  and  $O$  lie between  $A$  and  $B$ , we are to consider the ratios in the first term of our equation as positive. If either  $F$  or  $O$  lie external to  $AB$  the corresponding ratio or ratios is to be considered negative. Similarly for points  $E$  and  $N$  with respect to segment  $CA$ . When these rules are observed our theorem, as stated, will always be correct.

It is to be noted that rays  $BE$  and  $CF$  may remain fixed while  $NO$  rotates about fixed point  $P$  and the above relationship holds. Otherwise,  $NO$  may remain fixed while  $P$  traverses  $NO$  and the relationship still holds.

As a simple illustration of the use of the above theorem, let  $P$  be the centroid of triangle  $ABC$  and let  $NPO$  revolve about the centroid. In this instance  $AF/FB = 1$  and  $AE/EC = 1$  whereby  $AF/FB \cdot BO/OA + AE/EC \cdot CN/NA = 1$  becomes  $BO/OA + CN/NA = 1$  for all positions of  $NPO$  when  $P$  is the centroid of triangle  $ABC$ . As explained above  $BO/OA$  is positive when  $O$  lies between  $A$  and  $B$  and negative if  $O$  lies on  $AB$  extended. In a similar fashion we determine whether  $CN/NA$  is positive or negative.

Again, let  $P$  be the incenter of triangle  $ABC$  around which  $NPO$  is allowed to rotate. Then from elementary geometry  $AF/FB = b/a$  and  $AE/EC = c/a$  where  $a$ ,  $b$ , and  $c$  are the sides opposite the ver-



tices  $A$ ,  $B$ , and  $C$  of triangle  $ABC$ . Accordingly, in this instance,  $b/a \cdot BO/OA + c/a \cdot CN/NA = 1$  for all positions of line  $NPO$ . Let us go further and assume that  $N$  is the midpoint of  $CA$ . Then our fundamental equation becomes  $b/a \cdot BO/OA + c/a \cdot 1 = 1$  from which  $BO/OA = a - c/b$ .

It is interesting to allow  $P$  to assume the position of the circumcenter, orthocenter, symmedian point, nine-point center, etc. of triangle  $ABC$  and to check the results obtained by allowing  $NPO$  to rotate about point  $P$ .

As previously stated  $NO$  may be fixed and  $P$  allowed to vary thereon. Suppose  $NO$  be the line joining the midpoints  $N$  and  $O$  of sides  $CA$  and  $AB$ . Then  $BO/OA = 1$  and  $CN/NA = 1$  and our theorem reduces to  $AF/FB + AE/EC = 1$  for all positions of  $P$  on  $NO$ .

Let  $P$  again be the incenter of triangle  $ABC$  and let  $N$  be isotomic to  $E$  on  $CA$ . This means that  $AE = NC$  or  $AE/EC = CN/NA = c/a$ . Construct  $NP$  to meet  $AB$  at  $O$ . Then  $AF/FB \cdot BO/OA + AE/EC \cdot CN/NA = 1$  becomes  $b/a \cdot BO/OA + c/a \cdot c/a = 1$  or  $BO/OA = a^2 - c^2/ab$ . Now consider the case where  $BE$  is the symmedian from  $B$  to  $CA$  and  $CF$  is the internal bisector of angle  $C$  with  $BE$  and  $CF$  meeting at  $P$ . From modern geometry  $AE/EC = c^2/a^2$  and  $AF/FB = b/a$ . Choose  $N$  as the midpoint of  $CA$ . Draw  $NP$  meeting  $AB$  at  $O$ . Under these conditions the fundamental equation becomes  $b/a \cdot BO/OA + c^2/a^2 \cdot 1 = 1$  and, as before,  $BO/OA = a^2 - c^2/ab$ . Thus we have two lines intersecting at point  $O$  on  $AB$ .

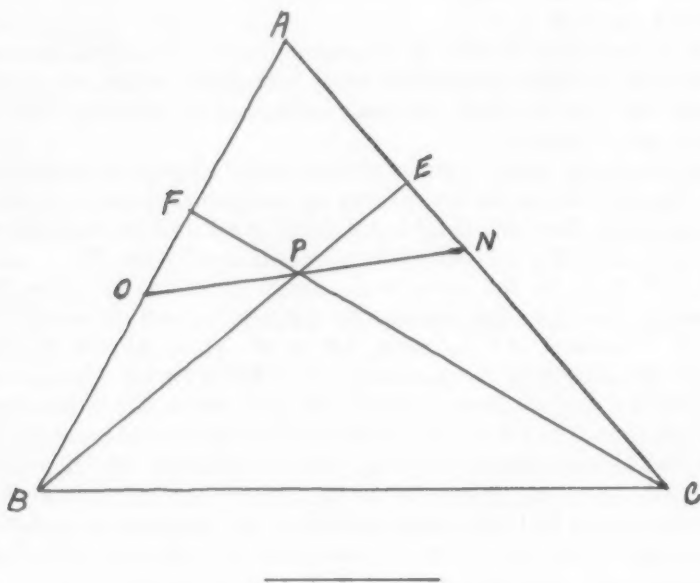
Finally, if we know two points through which the line  $NO$  passes, we may determine the position of points  $N$  and  $O$  on sides  $CA$  and  $AB$ . For example, let  $NO$  pass through the centroid and the incenter of triangle  $ABC$ . Since  $NO$  passes through the centroid, our theorem becomes  $BO/OA + CN/NA = 1$ . Also  $NO$  passes through the incenter from which we have  $b/a \cdot BO/OA + c/a \cdot CN/NA = 1$ . Solving these two equations simultaneously for  $BO/OA$  and  $CN/NA$ , we get  $BO/OA = a - c/b - c$  and  $CN/NA = a - b/c - b$ . If  $NO$  passes through the centroid and symmedian point, we find that  $BO/OA = a^2 - c^2/b^2 - c^2$  and  $CN/NA = a^2 - b^2/c^2 - b^2$ .

The so-called Euler line passing through the orthocenter, nine-point center, centroid, and circumcenter of a triangle may be shown to meet  $AB$  and  $CA$  in a manner such that

$$\frac{BO}{OA} = \left( \frac{a^2 - c^2}{b^2 - c^2} \right) \left( \frac{a^2 + c^2 - b^2}{b^2 + c^2 - a^2} \right) \text{ and } \frac{CN}{NA} = \left( \frac{a^2 - b^2}{c^2 - b^2} \right) \left( \frac{a^2 + b^2 - c^2}{b^2 + c^2 - a^2} \right).$$

It is to be hoped that the preceding examples will suffice to show the importance of the theorem. The author discovered its truth a number of years ago and with its use has been able to record a multi-

tude of important facts concerning various rays and lines associated with the triangle. It is to be hoped that the interested reader may utilize it for the same purpose.



#### POLIO VIRUSES PHOTOGRAPHED FOR FIRST TIME INSIDE CELL

Polio viruses have been seen and photographed inside the human cell in which they were formed.

This is the first time this has ever been done, the American Cancer Society reported.

While the researchers expect that this basic study may help explain how virus infections and symptoms develop—and how infections may be blocked—some mistaken ideas on viruses have already been cleared up. The following discoveries have been reported:

1. Viruses are formed in the cytoplasm surrounding the nucleus, not in the nucleus of the cell.

2. Some 100,000 viruses, occupying only about one or two percent of the cell volume, are produced in a cell during a few hours. (It would take almost 1,000,000 polio viruses lying side by side to equal one inch).

3. The tiny polio virus clusters in pure crystals inside the cell. Although scientists have previously produced crystals of viruses outside the cell and it has recently been shown that the large human herpes and adenoviruses can exist as crystals inside the cell, this is the first demonstration of small viruses of any kind inside the cell.

Scientists are now attempting to take electron micrographs—photographs with an electron microscope—of the actual process of virus manufacture inside the cell.

## The New Science and Mathematics\*

John R. Mayor

*AAAS and University of Maryland*

As a constructive result of a widespread concern, and sometimes unfair criticism, about the quality of science and mathematics teaching in the secondary schools for the past three or four years, very considerable progress is now being made toward real improvement. The great interest of school administrators, scientists, and teachers in modernizing secondary school courses in science is now manifested by much work in a variety of current science education activities.

*Curriculum Studies.* Perhaps the most encouraging sign of all is the importance of the work being carried on through the various curriculum studies in science involving scientists and teachers. For the most part, these studies are sponsored by the National Science Foundation, a federal government agency responsible directly to the President of the United States; but others, particularly in mathematics, have also received support from private foundations, especially from the Carnegie Corporation of New York. Now under way are major national curriculum studies in the sciences, including physics, mathematics, biology, geology, and chemistry. At the end of this paper is included a list of these studies and addresses from which additional information can be obtained. The best known of the projects, since it has been in operation for a longer period, is the work of the Physical Science Study Committee, centered at Massachusetts Institute of Technology. The School Mathematics Study group this year is sponsoring the try-out of sample textbooks for grades 7, 8, 9, 10, 11, and 12 by more than 600 teachers in some 150 schools in all parts of the country. In this activity the School Mathematics Study Group is demonstrating the great value to be obtained by teams of scientists and science teachers at the secondary level working together.

*Teacher Education and Certification.* During the past year, as many as 15,000 secondary school science and mathematics teachers have attended institutes or held fellowships provided by the National Science Foundation. This good work has been supplemented by support of summer programs by such industries as the General Electric Company and the Shell Oil Corporation. The institutes provide stipends for teachers of science and mathematics to study modern developments in science and mathematics which are particularly relevant to teaching in the secondary schools. One of the good features of the

\* Abstracted from a luncheon address given at the annual meeting of the Central Association of Science and Mathematics Teachers, Chicago, Illinois, November 28, 1959.

institutes has been the increased practice of including a demonstration class of high school students as a part of the total institute program. The institutes and regular college classes are making good use of the more adequate classroom materials for teachers, for group and individual study, which are being produced by the NSF curriculum studies.

Scientific groups, such as the American Association for the Advancement of Science and the National Academy of Sciences—National Research Council, have worked with the National Commission on Teacher Education and Professional Standards of the National Education Association in bringing together scientists and other academicians and professional educators in the improvement of teacher education programs at all levels. A recent promising enterprise, sponsored by the Carnegie Corporation of New York, is a study of teacher certification in science and mathematics, to be conducted by the National Association of State Directors of Teacher Education and Certification with the cooperation of AAAS.

*Consultant Services.* The National Defense Education Act of 1958, administered by the U. S. Office of Education, has provided funds which may be used by schools to purchase equipment for the teaching of science, mathematics, and the modern foreign languages. The Council of Chief State School Officers, with the assistance of representatives of such scientific societies as the American Institute of Physics, the American Institute of Biological Sciences, and American Association for the Advancement of Science has published a very comprehensive *Purchase Guide* for use by schools and teachers in purchasing science and mathematics teaching equipment. In addition the state departments of education in the 50 states have taken advantage of the provisions of the Act for financial assistance in the employment of supervisors in these three major areas of study. Practically all of the states have two or more supervisors in this field, now working with teachers in the schools. Related to consultant services to teachers is a study on the use of special teachers of science and mathematics in the elementary school, sponsored by the AAAS Science Teaching Improvement Program. This study is developed with a careful research design which should make it possible to provide objective evidence in support of special teachers or in support of the self-contained classroom.

*A New Partnership.* A most encouraging aspect of all of these efforts directed toward the improvement of science teaching is the evidence of a growing spirit of partnership and good will on the part of persons who often have been assumed to hold differences of opinion. This evidence is provided by such activities as curriculum studies, the cooperative work of groups interested in teacher education, the

purchase guide of the chief state school officers, and such projects as the use of special teachers in grades 5 and 6 working with science consultants. One of the most hopeful ways in which real improvement in education and in science education in particular can come is from the development of better working relationships among teachers at all levels of instruction and in all fields.

A further evidence of a going partnership is given by the recent affiliation of the Central Association of Science and Mathematics Teachers with the American Association for the Advancement of Science. As a staff member of AAAS I am most happy for this opportunity to welcome the members of CASMT among the affiliated societies of AAAS. The two organizations, one with emphasis on science and the advancement of science and the other with emphasis on teaching, are the two major overall science groups in this country. They have many common interests and the partnership should be a most fruitful one.

In these troubled times all of us have periods of being fearful for the future. Every person in this room wants very much to make a contribution to a better future, for all of the boys and girls, and babies yet to be born, in this country and in the world. I hold out to you additional hope, through the optimism expressed in terms of recent developments and the new partnership in the improvement of science, that you *will* make a major contribution to the improvement of the world tomorrow, if in your teaching you can develop in the boys and girls in your classes the inquiring mind, the desire to know the reason why, and the desire to pursue investigations in the scientific spirit. This is your challenge and your opportunity.

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NATIONAL SCIENCE FOUNDATION SPONSORED  
SCIENCE CURRICULUM STUDIES

- Physical Science Study Committee—Jerrold Zacharias, Director  
164 Main Street, Watertown 72, Massachusetts  
School Mathematics Study Group—E. G. Begle, Director  
Drawer 2502A, Yale Station, New Haven, Connecticut  
Biological Sciences Curriculum Study—Arnold B. Grobman, Director  
University of Colorado, Boulder, Colorado  
Teaching Resources Development in Geology—Robert L. Heller, Director  
University of Minnesota, Duluth Branch, Duluth, Minnesota  
Chemical Bond Approach Project—Laurence E. Strong, Director  
Department of Chemistry, Earlham College, Richmond, Indiana  
Chemistry Curriculum Study—Glenn T. Seaborg, Director  
University of California, Berkeley, California

## On the Equation $ax + by = c$ .

D. Mazkewitsch, University of Cincinnati  
Cincinnati, Ohio

1. Let  $a, b, c$  be positive integers,  $b > a$  and  $a$  prime to  $b$ . We want to find positive integral solutions of the above equation.  $c$  and  $b$  may be represented as

$$c = aq_1 + r_1; \quad r_1 < a \therefore \frac{r_1}{a} < q_1$$

$$b = aq_2 + r_2; \quad r_2 < a \therefore \frac{r_2}{a} < q_2,$$

where  $q_1$  is the quotient of the division of  $c$  by  $a$  and  $r_1$  the remainder of the division, similarly for  $q_2$  and  $r_2$ .

From  $ax + by = c$

$$\therefore x = \frac{c - by}{a} = \frac{aq_1 + r_1 - aq_2y - r_2y}{a} = q_1 - q_2y + \frac{r_1 - r_2y}{a}.$$

Since  $q_1$  and  $q_2$  are integers, and  $y$  has to be an integer,  $q_1 - q_2y$  is an integer. For  $x$  to be an integer  $y$  has to be selected so that  $r_1 - r_2y/a$  is an integer. For  $x$  to be positive we must have  $q_1 - q_2y > r_1 - r_2y/a$ . The upper limit of values which  $y$  must take is given by

$$c - by > 0 \therefore y < \frac{c}{b} \quad (\text{integer}). \quad (1)$$

If  $y$  is not too easy to evaluate from  $r_1 - r_2y/a$ , then put  $r_1 - r_2y/a = k = \text{integer}$ .

$$\therefore y = \frac{r_1 - ka}{r_2} = q_3 - q_4k + \frac{r_3 - r_4k}{r_2}.$$

Example. Find all positive integral solutions of the equation

$$13x + 17y = 3000. \quad 3000 = 13 \cdot 230 + 10; \quad 17 = 13 \cdot 1 + 4$$

$$q_1 = 230; \quad r_1 = 10 \quad q_2 = 1; \quad r_2 = 4.$$

Then

$$x = \frac{3000 - 17y}{13} = 230 - y + \frac{10 - 4y}{13}.$$

From the condition  $17y < 3000 \therefore y < 300/17$  or  $y < 177$ . Assume we do not see the value of  $y$  making  $10 - 4y/13$  an integer. Then let



$$\frac{10-4y}{13}=k$$

$$\therefore y = \frac{10-13k}{4} = 2-3k + \frac{2-k}{4} < 177.$$

$$\therefore k > -54. \quad (2)$$

For  $y$  to be a positive integer  $k$  must be negative and  $2-k/4$  an integer, the maximum value of  $k$  is evidently  $-2$ . The values of  $k$  form an arithmetic progression, beginning with  $-2$  and having a difference  $-4$ , then the last term of the progression is  $-50$  (because of (2)). Then  $-50 = -2 - 4(n-1) \therefore n = 13$ .

Hence there are 13 values of  $k$  and hence 13 positive solutions. They are

$$k = -2; \quad y = 2 + 6 + 1 = 9; \quad x = 230 - 9 + \frac{10-36}{13} = 219.$$

$$k = -6; \quad y = 2 + 18 + 2 = 22; \quad x = 202$$

$$\vdots$$

$$k = -46; \quad y = 152; \quad x = 32$$

$$k = -50; \quad y = 165; \quad x = 15.$$

2. In solving  $ax+by=c$  for integral solutions the following cases may occur.

I.  $c = a+b$ . Then the equation has evidently one pair of positive integral solutions:  $x=1, y=1$ .

II.  $c < a+b$ . Then the equation has no pair of positive solutions.

III. If in  $ax+by=c$ ,  $b > a$ ,  $c \geq ab$ , then there are positive solutions.

Proof. let  $c = ab + r$

$$\therefore ax+by = ab+r \therefore x = \frac{ab+r-by}{a} = b + \frac{r-by}{a}.$$

We can always find a positive value of  $y$  for which  $r-by/a$  is an integer and  $b+r-by/a$  will be positive. For, if, with  $y$  positive,  $r-by/a$  is positive, then  $x = b+r-by/a$  is evidently positive. If  $r-by/a < 0$ , then  $by-r/a > 0$ , and  $x = b-by-r/a$ . Now, from (1):  $by < c$ , or  $by < ab+r$

$$\therefore \frac{by-r}{a} < b \quad \left( \text{If } r=0, \text{ i.e. } c=ab, \text{ then } \frac{b}{a} < b \right)$$

and  $x$  is positive.

IV. In all other cases, if  $a+b < c < ab$ , there may be positive and negative solutions as the two following examples show.

$$1. 65x + 77y = 2000$$

$$x = \frac{2000 - 77y}{65} = 30 - y + \frac{50 - 12y}{65}$$

$$k = \frac{50 - 12y}{65} \therefore y = \frac{50 - 65k}{12} = 4 - 5k + \frac{2 - 5k}{12}$$

For  $k = -2 \therefore y = 4 + 10 + 1 = 15$  and  $x = 30 - 15 - 2 = 13$ .

$$2. 65x + 77y = 1000$$

$$x = \frac{1000 - 77y}{65} = 15 - y + \frac{25 - 12y}{65}$$

$$k = \frac{25 - 12y}{65} \therefore y = \frac{25 - 65k}{12} = 2 - 5k + \frac{1 - 5k}{12}$$

For  $y$  to be positive the maximum negative value of  $k$  is  $-7$

$$\therefore y = 2 + 35 + 3 = 40; \quad x = -32.$$

V.  $a$  and  $b$  have a highest common factor  $k$ . Then for the equation to have positive integral solutions,  $k$  must be a factor of  $c$ . After canceling  $k$  we obtain  $a'x + b'y = c'$ . According to III we must have  $c' \geq a'b'$ .

Example.  $8x + 12y = 376$ ,  $y = 2$ ,  $x = 44$ .

#### SEAWEED COMPOUND SUBSTITUTES FOR WHOLE BLOOD IN EMERGENCY

A seaweed compound mixed with water can substitute for whole blood in transfusions, two Japanese surgeons reported. Solutions of the gelatin-like substance which comes from the cells of the giant brown seaweed have been successfully used in 102 abdominal operations.

The use of "alginon," as the compound is called, is based largely on earlier research with another seaweed derivative called "algin" from sodium alginate. Both compounds have been tested as whole blood substitutes because they are made up of large proteins molecules or polymers. Like natural blood plasma, solutions of the seaweed compounds stay inside the blood vessels, keeping blood pressure from dropping to dangerous levels.

The new seaweed compound alginon does not damage the spleen, nor does it cause hemorrhages in the skin.

In tests with rabbits that had suffered bad burns or had lost a lot of blood—both shock conditions—no harmful changes in physiological processes were detected. Autopsies showed no effects on spleen, liver, kidney, adrenal gland, lung or brain tissues.

Furthermore, 70% of the alginon was excreted within 24 hours, indicating that alginon had a proper period of retention in the blood without depositing itself in tissue. It is better than the sugar- or salt-water solutions sometimes used in emergency transfusions because it is chemically stable, it improves the blood's ability to coagulate, and it does not dilute the capillary blood so that few red cells reach the tissues of extremities such as fingers or toes.

Unlike earlier versions of sodium alginate, this seaweed compound is said to cause virtually no harm.

## Requirements in Geology Departments

Roger H. Charlier and Courtland J. Daley<sup>1</sup>

University of Minnesota, Minneapolis, Minnesota

Claims have repeatedly been made that an extreme divergence of programs exists, throughout this country, in the various departments of Geology. These statements prompted the writers to embark on a survey of programs of institutions in the United States and Canada.

A questionnaire was sent out explaining that the aim of the survey was to devise a "standard" program for students majoring in geology, so that they may, *one* qualify for admission to major institutions offering graduate study, and *two*, prepare them for immediate employment in private industry, federal, state or municipal government agencies.

Rather than to devise a program which they felt would answer those needs, the writers outlined, in the questionnaire, the program of a typical college and invited comments from the departments surveyed. No inference was made that the program outlined did fulfill the aims set forth: the questionnaire was presented objectively, devoid of any comments. Forms were sent out to some 78 representative schools in the United States and Canada offering undergraduate programs and they included all schools in these countries offering a graduate program. The selection of the institutions was made from AGI Report 11, 1956-57 Edition, *Departments of Geological Sciences in the Educational Institutions of the United States and Canada*.

Questions were grouped under three headings: *Group A* listed all courses students were required to complete and *Group B* elective courses from which to choose so that the total number of credit hours earned in geology courses total exactly 32 hours (Table 1). All courses are semester courses of an average credit value of 3 semester hours.

Correspondents were asked to indicate which courses, among those listed under *Group A*, they felt should not be required, and which courses, among those listed under *Group B*, in their opinion should be added to the required subjects. In addition direct comment was invited by asking which courses not listed in either group, the respondent would expect a candidate for employment or for admission to graduate school to have completed in order to qualify.

The second part of the poll dealt with general degree requirements in (a) the other science courses (Table 1. Heading II) and in (b) non-science courses (Table 1. Heading III).

The questionnaire asked that the respondent indicate *first* which

<sup>1</sup> The writers were, until a few years ago, chairman and faculty member at a New York State college. Roger Charlier is currently at the University of Minn. and Courtland Daley became chairman of the Science Department, Cranford (NJ) High School.

TABLE 1  
AFFIRMATIVE ANSWERS

Subject	Heading	Yes answers	Code
Physical Geology	Group A	58	—
Historical Geology	id	58	—
Structural Geology	id	57	a
Mineralogy	id	57	b
Petrology	id	54	c
Sedimentation (*)	id	49	d
Stratigraphy (*)	id	56	d
Invertebrate Paleontology (#)	id	57	dd
Vertebrate Paleontology (#)	id	20	dd
Laboratory in Earth Sciences	Group B	26	e
Regional World Geography	id	3	e
Princ. of Geogr. (& Geogr. US)	id	5	e, h
Geogr. & Geol. of NY State	id	2	f
Field Geology	id	49	g
Geomorphology	id	30	h
Seminar	id	4	i
Optical Mineralogy	id	29	i
Economic Geology	id	27	i, j
Petroleum Geology	id	11	i, j
Senior Seminar	id	11	k
Workshop in Geogr. & Geol. of West. }	id	2	e, f, g
Europe (Field)			
Mathematics (2 courses)	Headg II	58	6, 10
Physics (id)	id	54	7
Inorganic Chemistry (2 sem.)	id	58	8
General Biology (id)	id	35	9, 10
Orientation	Headg III	30	—
English (12 hrs)	id	55	1, 10
Foreign Language (12 hrs)	id	51	2
History (12 hrs)	id	49	3
Psychology or Philosophy (6)	id	48	4
Music (6 hrs)	id	40	5

*Code explanations*

- (\*) either one of these two courses.  
 (#) id.  
 (a) one institution requires a course in advanced geology; one institution requires a course in advanced structural geology.  
 (b) one institution requires a course in crystallography; one institution requires a course in mineralography.  
 (c) five institutions require a course in petrography.  
 (d) nine institutions require both the courses in sedimentation and stratigraphy.  
 (dd) one institution requires both the courses in invertebrate and vertebrate paleontology.  
 (e) three institutions require a course in geography not listed in our questionnaire.  
 (f) these institutions require a course in the geology of their State; one institution requires a course in the geology of North America.  
 (g) twelve institutions require a field camp (Summer).  
 (h) one institution requires a course in the principles of geography or in geomorphology.  
 (i) one institution requires two courses marked (i)  
 (j) three institutions require either economic geology or petroleum geology; three institutions require 1 course in economic geology.  
 (k) one institution requires a course in conservation of natural resources; one institution requires a course in

courses in his opinion should not be required from candidates for employment or graduate study and *second* which courses he would like to see added, or substituted for courses deleted. Concreteness was invited, e.g. not 6 hours in mathematics but 3 hours in calculus and 3 hours in statistics. For the non-science courses the respondents were asked to endorse or correct the number of semester hours required under the tested program and to add, delete or substitute if necessary.

To the 78 questionnaires sent out, we received 58 answers or 74.35%.

Table 1 lists the courses of Groups A and B and those of headings II and III with the number of affirmative answers received. Affirmative answers are to be interpreted as endorsements. For courses listed under Group B affirmative answers mean that the replying institution feels the specific course should be made a requirement. No effort has been made to list the number of credit points in geology required in each institution, since in most cases this matter was not indicated by the respondent; this could make another illuminating poll.

The authors took then the 58 responses as a 100% basis and computed the percentage of affirmative answers which is illustrated in Table 2. The table gives at a glance an idea of similarities and divergences in programs throughout the U. S. and Canada.

It appears from the above returns that in general courses from *Group A* are universally accepted and required, with, of course, a slight occasional variation from institution to institution. However, most institutions required more courses, considered Field Geology as belonging in *Group A* rather than in *Group B* and felt a required Summer Camp an essential part of undergraduate geological preparation. Summer Camp is indeed rapidly gaining acceptance in all institutions.

photogeology; one institution requires a course in correlation; two institutions require a course in geophysics; two institutions require a course in geological report writing; two institutions require a course in plane table work; two institutions require a course in mapping; one institution requires a course in geochemistry; one institution requires a course in geological methods; two institutions require a course in either aerial photogeography or aerial photogeology; one institution requires a period of geological employment; one institution requires a course in geophysical prospecting.

(1) five institutions recommend reduction; 1 recommends a course in English composition; 1 recommends a course in scientific English.

(2) two institutions recommend increase in number of hours; two institutions recommend decrease.

(3) three institutions recommend decrease in number of hours.

(4) four institutions recommend decrease in the number of hours.

(5) five institutions recommend decrease in number of hours.

(6) 35 institutions require calculus in addition to or as part of 12 hours of mathematics; 5 institutions require descriptive geometry in addition to or as part of 12 hours; 3 institutions require analytical geometry in addition to or as part of 12 hours.

(7) 7 institutions require increased preparation in physics.

(8) 9 institutions require physical chemistry; two require increased preparation in chemistry.

(9) 11 institutions require qualitative analysis; 9 institutions require quantitative analysis; 3 institutions require a course in "analytic" chemistry.

(10) 2 institutions require a course in public speaking; 3 institutions require a course in engineering; 1 institution requires a course in technical drawing; 4 institutions require a course in mechanical drawing; 2 institutions require a course in cartography; 3 institutions require a course in statistics; 5 institutions require a course in surveying; 1 institution requires a course in engineering drafting.

TABLE 2  
PERCENTAGE OF AFFIRMATIVE ANSWERS

Geology Courses	%	Non-Geology Courses	%
Physical geology	100	Inorganic chemistry	100
Historical geology	100	Bio-chemistry	60.3
Structural geology	98.2		
Mineralogy	98.2		
Petrology	93	English	94.8
Sedimentation	84	Foreign language	87.9
Stratigraphy	96.5		
Invertebrate paleontology	98.2		
Vertebrate paleontology	34	Psychology	82.7
Laboratory in Earth Sciences	44		
Regional World Geography	5	Music	69
Princ. of geogr. (& US geography)	8		
Geology & Geography of NY State	3.4		
Field geology	84.4	Mathematics	100
Geomorphology	51.2	Physics	93.1
Seminar	6.8		
Optical mineralogy	3.4		
Economic geology	46		
Petroleum geology	17		
Senior seminar	17		
Geol. & Geogr. W. Europe (Field)	3.4		

The authors, for once departing from their "objective" attitude, feel personally inclined to consider these courses as of great importance.

Seven institutions disapprove of a B.A. degree and want the candidate for graduate study to present himself with a B.S. degree. This is not the place to discuss in detail this important problem.

Except for optical mineralogy, economic geology and geomorphology, graduate schools wish to cater themselves to the analytical preparation and specialization of the future senior geologist.

Non-geology requirements seem pretty well uniform, but show a pronounced tendency to prefer analytical chemistry instead of biology, unless the student is preparing for specialization in paleontology. A further conclusion easily drawn from the collected data is that calculus should be made a requirement for those students planning to enter graduate school and the authors strongly recommend that subject for all geology majors. Attention should also be given to at least an introductory course in statistics including the computation of the four moments since there is a marked growing interest in statistical computations in the field of sedimentation.

We have been humored a good deal about the course in "Orientation." Deletion of such course is generally recommended, except if it is orientation to the geological sciences. The authors enjoyed the off the cuff comments which cheered them up each time collating became tedious.



Our sincere gratitude goes to all the departments who so willingly gave their time to answer a rather lengthy questionnaire.

Many questionnaires were returned with additional comments pertaining to curriculum and preparation of students in general, some of them going back to the secondary level. Graduate schools complain often about the insufficient preparation of products of the liberal arts college; but, have we ever asked ourselves whether the solution does not lie at the high school level?

Earth Sciences, a convenient name designed to avoid ruffling the feathers of either geologists or geographers, and permitting to include some notions of astronomy, cosmography and meteorology, have been surprisingly neglected at the high school level. Some states—such as New York, Pennsylvania, Ohio, Michigan—have recently added—or revived—the course to the high school curriculum (*Moss, Pollack, Hill*). Minnesota offers a 2-year sequence at the University High School: this course might very well put freshman geology out of business.

Implementation of an Earth Science course is not overly simple since there is a lack of qualified personnel: "most school science teachers are poorly equipped to teach earth science" and, as is true with other sciences, "almost universally any graduate work of high school science teachers has been done in the professional field of education" (*Brown*). Yet, the new course has proven most successful in Pennsylvania schools where 13 weeks are spent on geology in grades 7 through 9 (*Moss*). Universities have offered special summer programs for high school students interested in geology; Columbia's School of Engineering, through a grant of the National Science Foundation and in collaboration with the university's department of geology, organized a special colloquium for talented high school pupils and the University of Oklahoma arranged for a challenging program (*Moss, Pollack*).

The Pennsylvania course deals for about half of the allotted time with geology and oceanography and the study of minerals and rocks has generated substantial enthusiasm. Conferences such as the one held last summer in Duluth help bring together the college and high school teachers, as well as field geologists, and bring about the necessary articulation among scholars and all level teachers.

The Oregon Museum of Science and Industry in Portland sponsors six junior science clubs and runs the first youth camp in this country, devoted to teaching geology and paleontology, in the fossil rich hills of eastern Oregon. The New Jersey Academy of Science, through its planned establishment of Junior Academies, will attempt to introduce high school youth to earth science.

The moral of our findings is that more and more advanced specialization makes it necessary for the undergraduate student to absorb a

continuously increasing amount of subject matter. We feel that at least the basic notions of earth science should be made part of the high school program and that the present system of 2- or 3-week-units is unsatisfactory. In doing so, our high schools would only be adopting practices followed for many years by their European and Brazilian counterparts (*Charlier, Schroeder, Sternberg*).

While one will not find in high school catalogues across the Atlantic a course in geology, or in earth sciences for that matter, geography appears as a required subject everywhere and for a total of at least 180 hours of actual class time. The definition of geography, however, differs considerably from the notion many of our teachers have of it; European geography follows closely the patterns of *Finch* and *Trewartha*, a blending, in equal parts, of the physical aspects of the earth and of the place and activities of humans thereupon. And so it comes that besides general ideas of commerce, land use, transportation, habitat and the like, the student learns as well about morphology, hydrography, petrology, mineralogy and soil composition. By the time he takes his first college course in geology, the professor has merely to refresh the student's memory in two or three lectures instead of spending one or two semesters in initiation and orientation (*Charlier*).

Moreover, the regional approach is used to apply these basic ideas to a variety of countries. High school students seem fascinated with learning not only about the people around them and beyond far away borders, but as well about the rocks and plains and mountains.

The earth, to all of us, should be an intimate living thing, of mood and character which like an old friend can be gauged and interpreted; the more we try to understand this friend the more beneficial this friendship will grow.

Earth Science, at high school level, offers an outstanding opportunity to show the utilitarian side of other sciences, the interdisciplinary link which we fail so often to underscore: physics find their application in structural studies and tectonics, chemistry in crystal shapes, mineralogical composition and metamorphism, botany and zoology are the very bases of some sedimentary processes, of coal and reef formations, of fossil study. In turn the land we live on, is a barometer of our economic and social well-being.

"Spectacular" examples of utilization of geological knowledge are as numerous as they are colorful: our petroleum industry just celebrated its centenary, Prince Rainier of Monaco just prohibited use of some roads by heavy trucks because of the lack of geological forethought when the roads were built, and the famous Riviera beach of Nice is one of pebbles because no geologist was consulted when the

*Promenade des Anglais* and the *Quai des Etats-Unis* were constructed. The industrial growth of our nation depends heavily on geology. The search for mineral resources including gas and oil, the development of mineral deposits, the occurrence and quality of our water supplies, the construction of tunnels, dams, highways, bridges, buildings, military strategy even are avenues of the geologist's endeavour. The Russians, who have a Cabinet post for geology, found out that even central heating installation can do with geological advice.\*

Often, resistance to introduction of any type of earth sciences course at the high school level has been due to the belief that such knowledge is a waste of time for girls. First of all, it has never hurt anyone to know something about the world he lives in and since frontiers are shrinking such knowledge no longer remains a luxury but is pretty well a necessity (*Grace*). But there is another rebuttal of this fallacy: with each passing year more women become successful geologists, geophysicists and the like. The range of their employment runs the entire gamut from teachers, research workers, writers, librarians to field geologists and executives. The incidence of husband-wife geologist teams is conspicuously high.

In geology, as well as in all other provinces of scientific endeavour, the amount of college work increases continuously, a normal consequence of the growing complexity of this science. The length of universities studies—to which military service for men has to be added—can hardly be prolonged. Undergraduate departments, blocked by overall college requirements for degrees, find it difficult to include all pertinent material in the total number of credit hours allotted for the "major." If high schools could offer the basic information in earth sciences, valuable time would be saved in college and for those students not considering a career in earth sciences, not planning even to elect this course as a fulfillment of their science requirement, or simply not intending to go to college at all, exposure to a panoramic view of the world around them, the people on it and the economic and social implications thereof, can only benefit citizens of a country upon which a major part of world leadership has been trusted.

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\* In Norilsk, Siberia a brand new workers apartment building was built on permafrost. The centrally located heating plant melted the upper layer of the permafrost, causing the building to tilt and sink at an angle. The lopsided building can now only be heated by individual space heaters in each room.

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## Who Gets Credit for Scientific Achievement?

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Cases abound of investigators who were ahead of their times. Peter the Stranger's work on magnetism foreshadowed William Gilbert's by several hundred years. Niels Stenson in 1669 wrote about the Graafian follicles before deGraaf. He suggested the use of fossils to trace the history of the earth about two hundred years before it was actually done. The Frenchman deMaupertuis who lived during Newtonian times anticipated the work of Mendel, Darwin, and deVries about 150 years ahead of them. In the nineteenth century, the English mathematician Babbage worked on computing machines about a hundred years before the twentieth century extensive development.

It is easy to plead that the earlier investigators who foreshadowed later triumphs were victims of their times. The people of their age were not ready for the notions and devices they promoted. But what apologies are available for the multitude of near misses in achievement by the contemporaries of celebrated scientists?

Cesalpino was very close to the discovery of the circulation of the blood. Is Harvey given credit because his description was more lucid? Only lately has the genius of Robert Hooke been found to have duplicated and indeed to have anticipated the work of Newton and Boyle. The latter sponsored Hooke and the air pump and Boyle's Law may be Hooke's in reality. Newton despised Hooke, and maybe in part because Hooke had a universal law of gravitation also.

Benjamin Franklin's work with lightning merits attention but what about Ruhmann of St. Petersburg? Was Franklin a better communicator? Skill at public relations should not be important in the assigning of credit for scientific work, yet the history of the law of conservation of energy is rich in this matter. Robert Mayer was a young physician sailing in the East Indies when he sent a paper to Poggen-dorff's *Annalen de Physik* stating the idea. Yet Helmholtz permitted himself to be called "the father" of the first law and James Joule sought credit for it.

In all accounts of the history of the periodic table of chemical elements, Dmitri Mendeleef is promoted as the originator. Yet Mendeleef could not have succeeded without the groundwork of Newlands, LaChancourtois, and Julius Lothar Meyer. The latter's table was used by Mendeleef who had the audacity to predict with it. If the aim of science be considered prediction, then Mendeleef has an edge, but if the aim of science be the establishment of universals, then Lothar Meyer also deserves to be applauded.

The periodic table of chemical elements is interrelated to the other achievements of chemistry and science; one is connected to the other. Without the atomic theory—who gets credit for that, John Dalton or as the Irish claim, Bryan Higgins?—and other notable advances of nineteenth century chemistry, the periodic table could not have been. We must perhaps modify Newton's statement that he stood on the shoulders of giants to realize that all researchers stand with hands outstretched to each other and to the cultural heritage of the past.

In the bitter debate over credit, hand grasping is difficult to detect. The great and generous Michael Faraday refused to admit William Sturgeon's priority in discovery. Both James Watt and Antoine Lavoisier, honored anyway, claimed to have found the compound nature of water. The discovery of anaesthesia has a full share of battle about priority.

The pleasant and civilized manner in which Charles Darwin and Alfred Russel Wallace settled the issue of credit seems to be an exception and yet it must in the future be the rule. As scientific research multiplies, the dependency of one investigator upon the other increases. In this time of celebrating Darwin's work it would be wise to emphasize the inter-relatedness of science and scientists.

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#### SCIENTIST MEASURES HEAT GULF STREAM GIVES NORWAY

How much heat the Gulf Stream gives off to Norway each year has been measured by a Norwegian scientist.

It is equal to the heat that would be produced by burning the amount of oil that could fill a 100,000-ton super-tanker every other minute for a full year.

This northernmost country in Europe is greatly benefited by the Gulf Stream, whose warm waters sweep along the western coast and keep harbors ice-free all winter long.

Dr. Hakon Mosby of the University of Bergen's Geophysical Institute made this approximation of Norway's indebtedness to the Gulf Stream largely from data collected at a weather station in the Norwegian Sea at 66 degrees north latitude and 2 degrees east longitude.

These data indicated that of the total heat loss from the surrounding waters, 34 kilocalories per square centimeter were given off to the atmosphere each year. (A kilocalorie is the amount of heat required to raise the temperature of one kilogram, or 2.2 pounds, of water one degree centigrade.)

This was roughly equivalent to the heat combustion of a layer of oil a little more than one inch (three centimeters) thick over the whole area of the Norwegian Sea, about 390,000 square miles.

This is as much oil as could be contained in a 100,000-ton super-tanker if it were loaded every other minute for a full year.

Comparable studies indicate that the Gulf Stream gives off in the Arctic Ocean only about one-fourth the heat it gives off in the Norwegian Sea.



## The Elementary Mathematics Program in Russia

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American awareness that recent Soviet achievements in space technology bespeak a high order of mathematical competence has aroused interest in the teaching of mathematics at all levels in Soviet schools. The subject is a complicated one, as it is indeed in our own country, and much more detailed study must be made before anything like a fully adequate picture can be obtained. As a contribution along these lines the present article offers a translation of the official syllabus for the first four grades of elementary school.

As usual with documents of this sort the language is replete with technical jargon and is frequently terse to the point of ambiguity. In order that the material may be as meaningful as possible to the reader I have provided fairly extensive notes wherever these seemed desirable. The notes are based largely on my examination of a number of elementary textbooks and works for the training of Soviet teachers. The main contents of this article, that is the three sections entitled "Explanatory Remarks," "Methodological Instructions," and "Programs," are translated from the official Russian school programs for 1957-58.<sup>1</sup>

### EXPLANATORY REMARKS

The teaching of arithmetic in Grades I-IV has as its aim to teach children correctly, knowingly, confidently, and rationally to perform operations with whole numbers and to apply acquired knowledge and experience to solving arithmetic problems and performing simple calculations. The teaching of arithmetic must help carry out the tasks of the Communist upbringing of children.

The study of arithmetic in school should be so designed that number and measurement serve as a tool of cognition of surrounding reality.

During the instructional period of Grades I-IV the pupils should acquire:

A firm knowledge of whole numbers and operations with them and solid habits of oral and written computation with whole numbers, both abstract and denominate:

A firm knowledge of metric measures and measures of time, and the ability to use them in measuring;

<sup>1</sup> *Programmy nachalnoi shkoly na 1957-58 uchebny god* (Moscow, 1957), pp. 70-81. For further information on the official syllabi and their relationship to the Soviet school system see Izaak Wirszup, "Current School Mathematics Curricula in the Soviet Union and other Communist Countries," *The Mathematics Teacher*, LII, 5 (May, 1959), p. 334 ff.

An elementary knowledge of simple fractions;

A knowledge of a few elements of descriptive geometry and the ability to apply this knowledge in practice;

The ability to solve easy arithmetic problems.

Whole numbers and operations with them, studied in a definite order, comprise the basic content of the program of Grades I-IV.

First of all the children study numeration and the arithmetic operations (addition and subtraction) within the limit of 10, and then numeration and the four arithmetic operations within the limit of 20. Next comes the section "Numeration and the four operations within the limit of 100," the study of which begins in Grade I and is concluded in Grade II, after which begins the study of numeration and operations within the limit of 1000. From this section only numeration and the four operations in round hundreds are studied in Grade II.

In Grade III oral computations with round tens within the limit of 1000 are studied, and written methods of calculation within this limit are introduced.

After the study of the operations within the limit of 1000, numeration and the four arithmetic operations involving large numbers within the limit of a million are studied.

Pupils learn the reading and writing of numbers and familiarize themselves with the designation of the orders and their distribution into periods within the limit of six-digit numbers.<sup>2</sup>

In Grade IV the knowledge which has been acquired by the pupils is expanded and systematized: the knowledge of numeration is expanded to include the periods of millions and billions, more difficult cases of multiplication (multiplication of numbers with zeroes at the end) and division (division with a remainder, etc.) are consolidated; the relationship between data and results of arithmetic operations is learned;<sup>3</sup> this last is used for checking operations, for solving exercises with  $x$ , and for solving problems.<sup>4</sup>

The course in primary arithmetic concludes with the study of simple fractions in Grade IV.

<sup>2</sup> "Orders" are the successive positions occupied by the digits of a number. For example, a digit in the one's place is called a digit of the first order, a digit in the ten's place is called a digit of the second order, and so on. "Periods" refers to the successive groups of three digits, which in our notational system (but not in the Russian) are set off by commas.

<sup>3</sup> As an example of what is meant by "the relationship between data and results" reference may be made to a textbook used in the fifth and sixth grades. (I. N. Shebchenko, *Arifmetika. Uchebnik dlya 5 i 6 klassov semilyetii i srednei shkoly*, [Moscow, 1958], pp. 49-50) Under this heading the author illustrates the relationship for addition by a discussion which concludes with the following note: "In order to find an unknown addend it is necessary to subtract the known addend from the sum of the two addends." This is further expressed with the notation that if  $a+b=c$ , then  $a=c-b$  and  $b=c-a$ .

<sup>4</sup> The word translated as "exercise" refers to a pure number problem, such as 18-13, whereas "problem" refers to what we frequently designate as "word problem." The expression "exercises with  $x$ " refers to such exercises as the following, taken from a fourth-grade Russian textbook: Find the value of  $x$  in  $x-625=1,200$ . (A. S. Pchylko & G. B. Polyak, *Arifmetika. Uchebnik dlya 4-go klassa nachalnoi shkoly*. [Moscow, 1956].)

As a result of their study of whole and fractional numbers, pupils who complete Grade IV should:

- Have a firm grasp of the terminology of each arithmetic operation;
- Have a good command of the technique of written computation;
- Know the formulations in which the relationship between the component parts of operations is expressed, and the rules for checking operations;

Understand the significance of each arithmetic operation (without memorization of specific operations) and know the basic circumstances in which each operation is applied to the solution of problems;

Be able to make use of the basic properties of operations in oral and written computations without the formulation of these properties, with the exception of the commutative property<sup>5</sup> of addition and multiplication, which pupils should be able to name and formulate;

Know the formation of the fractions ( $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{1}{16}$ ) and how to add and subtract fractions having the same denominator and denominators which are multiples of each other.

Throughout every year of the primary arithmetic course great attention is devoted to the study of metric measurement, and also measurement of time.

So that the students will obtain a concrete idea of all the measures and learn to use them, in every grade, beginning with the first, the children themselves make up examples of various measures and practice measuring and weighing, developing judgment of eye, and determining approximate weight of solids by muscular sensation of weight.

In Grade IV the operations with compound denominate numbers<sup>6</sup> should be limited to the simplest cases of these operations with small, two-digit numbers to whatever extent is required for preparing children to study decimal fractions and for practical use in life. In particular, operations with numbers which express measures of time should be made easier.

A substantial part of the program of primary arithmetic is made up of geometric material, the study of which gives some knowledge and experience to the children and develops spatial concepts in them. The study of this material begins in Grade I and continues throughout the whole course of primary school, becoming gradually more complicated and closely interwoven with arithmetic. In Grades I and II, in the study of numeration and operations with numbers, squares, rectangles, triangles, circles, cubes, and rectangular solids are employed as didactic material. The children obtain visual images of these figures and bodies, become familiar with their names, draw and

<sup>5</sup> The principle that  $a+b=b+a$ ,  $ab=ba$ .

<sup>6</sup> Compound denominate numbers refer to magnitudes expressed in units of more than one denomination. *as* 2 meters 15 centimeters.

trace them on graph paper, and measure the length and width of rectangular figures.

Many exercises are carried out by children in the measurement of segments of a straight line with the aid of a centimeter ruler and of distances with the aid of a meter or a tape measure.

In Grade III children obtain practice in more difficult measurement of segments of a straight line and of dimensions of small objects, expressing the results of the measurements in decimeters, centimeters, and millimeters. Measurements are carried out not only in the classroom, but also out in the open, where children learn to stake out and measure straight lines and gain practice in the development of visual estimation.

In Grade IV the pupils become acquainted with square meters and with the calculation of areas having rectangular form.

For the study of these subjects practical exercises are provided in measuring the area of the classroom floor, the illuminated area of the class, the area of the school garden and of the schoolyard, and so on.

In the fall and spring pupils become acquainted visually with the are and the hectare on location, and also with the construction of a right angle, a square, and a rectangle.

After learning square measure and calculating area the pupils study cubic measure and learn to calculate the volume of bodies having right angular form (boxes, rooms). This must be limited to the solution of problems in which the dimensions of the sides are given and it is necessary to find the area or the volume.

In the calculation of area and volume a notation is used which is based upon the method of measurement of these magnitudes and is easier for students in the primary grades. (An example of this notation is  $6 \text{ sq. m} \times 4 = 24 \text{ sq. m.}$ )

About half of the time devoted to arithmetic in classwork and homework should be used for teaching children the solution of arithmetic problems.

The ability to solve arithmetic problems is one of the basic aspects of the general-educational significance of the arithmetic course. The solution of problems fosters the development of the students' thinking and speaking, their discrimination, and their ability to determine the relationship among magnitudes and to make correct inferences; the solution of problems helps to prepare students for life.

The solution of problems helps the students to understand the concrete meaning of arithmetic operations, elucidates the diverse circumstances in which they are applied, provides elementary practice in the application of analysis and synthesis.

Beginning with the first steps in teaching and extending throughout the whole course of arithmetic, the solution of problems proceeds

parallel with the teaching of operations.

In teaching the solution of problems it is necessary to adhere to a precise sequence in going from easy problems to more difficult ones, from the simple to the complex.

In Grades I, II, and III the pupils become acquainted with the basic aspects of simple problems in each of the four arithmetic operations.

In Grade I simple problems are solved in finding the sum and the difference, in increasing and decreasing given numbers, in finding the product (when a given number is repeated as an addend several times), and in partitive division.<sup>7</sup>

In Grade II problems are solved in comparison by subtraction,<sup>8</sup> in finding one of the addends from the sum of two numbers and one of the numbers, in finding the minuend from a given subtrahend and difference, in measurement division, in increasing and decreasing given numbers by "so many" times,<sup>9</sup> in finding the fractional parts of a number, and in comparison by division.<sup>10</sup>

In Grade III, to supplement what was indicated above, simple problems are solved in finding the subtrahend from a given minuend and difference.

In addition to the simple arithmetic problems, composite problems of gradually increasing complexity are solved in primary school, beginning with Grade I.

In Grade II along with others are solved problems in direct and inverse reduction to a unit.<sup>11</sup> Among the composite problems are distinguished ordinary arithmetic problems, which are solved in close connection with the study of the arithmetic operations, and the so-called typical problems;<sup>12</sup> the latter are introduced only in Grades III and IV.

<sup>7</sup> Partitive division refers to finding the size of the unit-group when given the whole group and the number of groups. For example, how many people are there in each group if 100 people are divided into 5 groups? It contrasts with measurement division, which consists of finding the number of groups when given the whole group and the size of the unit-group. For example, how many groups are there if 100 people are divided into groups of 20 people?

<sup>8</sup> Literally, "difference comparison." Under this heading a Grade II text depicts two rectangular bars, one 7 centimeters long and the other 4 centimeters with the notation that one is "3 centimeters longer." (A. S. Pchyolko & G. B. Polyak, *Arifmetika. Uchebnik dlya 2-go klassa nachalnoi shkoly*, [Moscow, 1956], p. 22).

<sup>9</sup> The Grade II text mentioned above deals with this type of problem in pages 52 ff. Under the heading "Increasing a number several times" it depicts 6 hammers in contrast with 3 hammers, and states that the former are two times as many as the latter. It stresses the contrast between this and the process of increasing a number by some unit-number, as in the case of 5 cherries versus 3 cherries; the former are 2 more than the latter. The section "Decreasing a number several times" includes a problem "8 mushrooms divided by 2 equals 4 mushrooms." This also is contrasted with the process of decreasing a number by some unit-number: "8 mushrooms less 2 mushrooms equals 6 mushrooms."

<sup>10</sup> Literally, "multiple comparison." Under this heading the work cited in Note 8 depicts (p. 74) two rectangular bars and notes that one is "3 times as long as" the other. [It may be well to note that the phrase *3 raza dlinnyshe* does not mean, as it might seem to at first glance, "3 times longer (than)," but rather "3 times as long (as)."] Further along (pp. 77-78), it is stated that "In order to find how many times as large or as small one number is as compared to another, we must divide the larger number by the smaller." Comparison by division and comparison by subtraction are contrasted with each other in the following type of exercise: "5 centimeters is how many times as small as 35 centimeters? 14 rubles is how much less than 60 rubles?"

For examinations it is necessary to select problems which do not go beyond the scheduled requirements.

Pupils should be able to do the following:

At the end of the first year of instruction: correctly write down the solution of problems and the results of operations, and explain them orally;

At the end of the second year of instruction: pose a question orally and name the operation for its solution, write down the operation correctly with a description of the components; coherently recount the successive steps in the solution of a problem after solving it (without prompting from the teacher);

At the end of the third year of instruction: briefly write down the conditions of a problem, independently formulate a plan for its solution, and write down the solution of the problem together with a written formulation of questions;

At the end of the fourth year of instruction: independently write down the conditions of a problem, coherently explain the steps in its solution and the choice of operations, precisely formulate and write down questions, and check the solution of simple problems.

In the program much attention is devoted to developing habits of *oral computation*.

In the course of the first two years of instruction the pupils use only oral methods of computation. Beginning with Grade III, written computations comprise the basic form of computation. However, work in familiarizing students with various methods of oral computation and in developing habits of fluent oral computation should continue up to the end of the course in arithmetic. In this connection special attention should be devoted to developing fluency in a computation within the limit of 100, and also with large numbers, operations with which reduce themselves to computation within the limit of 100, for example,  $120 \times 3 = 12 \text{ tens} \times 3$ ;  $480 \div 6 = 48 \text{ tens} \div 6$ ;  $25,000 + 36,000 = 25 \text{ thous.} + 36 \text{ thous.}$

<sup>11</sup> As an example of "direct reduction to a unit" we can cite, from p. 46 of the Grade II work mentioned above, the problem "4 pens cost 20 kopeks. How much were 3 such pens?" The steps in the solution of this are given as:

$$(1) 20\text{k} \div 4 = 5\text{k}$$

$$(2) 5\text{k} \times 3 = 15\text{k}$$

As an example of "inverse reduction to a unit" the same work (p. 68) gives the problem "3 dresses were sewn from 12 meters of material. How many such dresses could be sewn from 20 meters of material?" Here the steps are:

$$(1) 12\text{m} \div 3 = 4\text{m}$$

$$(2) 20\text{m} \div 4\text{m} = 5$$

<sup>12</sup> These are problems which are "typical" or "characteristic" of "special methods" of solution, such as proportional division, sum and ratio, rule of three, and others. In one collection of problems and exercises for fourth grade, "typical problems" for each of the special methods are listed by number at the beginning of the work but are scattered throughout the book under various topics. (See the 1946 and 1950 editions of N. N. Nikitin, G. B. Polyak, and L. N. Volodina, *Sbornik arifmeticheskikh zadach i uprazhneniy dlya chetyortogo klassa nachalnoi shkoloy*).



It is necessary first to strive for mastery by the students of methods of oral computation, and afterwards to acquaint them with special (short) methods of oral computation, devoting special attention to those methods which lead to easy and rapid computation; rounding, commutation of addends and of multiplicand and multiplier, successive multiplication and division in simple cases (e.g. multiplication and division by 2, 4, 8),<sup>13</sup> and short multiplication by 5, 25, 50.<sup>14</sup> The practice in oral computation within the capabilities of the children should involve not only abstract numbers but also compound denominate numbers, not only exercises but also problems to be worked out. Much attention should be devoted to the solution of oral problems.

Along with oral and written computations it is necessary to teach children computation on the abacus, which, as is known, finds wide application in living practice.

#### METHODOLOGICAL INSTRUCTIONS

Arithmetic teaches quantitative relationships of the real world. For the teaching of arithmetic to contribute to the correct reflection of these relationships in the consciousness of children, it must be closely tied in with life, with reality.

A pupil can choose correctly the operation for the solution of a problem only if he knows what connection exists among the magnitudes which are mentioned in the problem. Hence in solving problems it is necessary to rely heavily on the living experience of children, to tie in closely the teaching of arithmetic with life.

The contents of problems should as much as possible reflect the laboring, productive reality of the people and should have a perceptive character.

Apart from the problems in arithmetic books, it is necessary in each class to solve problems based on numerical data taken directly from surrounding life, from local production familiar to the students.

It is necessary step by step to train children in the independent creation of problems analogous to those solved in class.

Students can gain practical experience in the solution of problems by carrying out simple calculations connected with some activity or other (excursions, school holidays, working on the student experimental plot of land, etc.). It is necessary to teach students to use reference works and tables given in the arithmetic book.

The study of arithmetic should help the child to achieve firm habits in computation and mensuration. The school should create firm com-

<sup>13</sup> E.g.  $75 \times 4 = 70 \times 4 + 5 \times 4 = 280 + 20 = 300$ , and  $75 \times 4 = 75 \times 2 \times 2 = 150 \times 2 = 300$ .

<sup>14</sup> E.g.  $63 \times 5 = 63 \times 10 \div 2$ .

putational habits by means of an adequate amount of practice in the solution of numerical exercises and problems, and also mensuration habits by means of practical work in measuring, weighing, etc. in class and on location. This work should be preceded by detailed explanations by the teacher so as to secure a well-grounded mastery of arithmetic by the students. Practice in the solution of problems and numerical exercises should be carried out not only in class but also at home. Independent written work by the students should in large measure serve the aim of creating firm habits.

It is necessary to advance to the development of abstract mathematical concepts in primary school by starting with visual instruction. Accordingly in the teaching of arithmetic wide use should be made of visual aids: arithmetic box,<sup>15</sup> class abacus, models of metric measurements, geometric figures, measuring and drawing instruments (ruler, compasses, and drawing-triangle and drawing-square<sup>16</sup>), simple instruments for land-measurement (surveying-square,<sup>17</sup> tape-measure or measuring chain), graphic illustrations.

The direct visual-tactile perception by the students contributes to the successful teaching of geometric material. The students should not only use ready-made graphic figures, given by the teacher, but also themselves make and reproduce geometric forms: model, cut, mount, draw, glue, and secure geometric figures by paper-folding.

It is important also to make use of self-made aids, as for example: for computation, a self-made abacus; for geometry, geometric figures; for measurement, models of measures.

The students themselves should be drawn into the preparation of these aids. The work of preparing visual aids helps children to achieve better mastery of those ideas which are illustrated with the help of the aids, and teaches students work habits and skills: cutting, mounting, sawing, planing, etc., using scissors, knife, saw, ruler, circle).

A great diversity of activity in arithmetic is provided by mathematical games and recreational problems, which can be used in the lessons of Grades I and II and in extracurricular activity in Grades III and IV.

<sup>15</sup> This is described by V. G. Chichigin in his *Metodika prepodaniya arifmetiki* (Moscow, 1952), p. 15, as a box filled with wooden cubes, rectangular solids, and plane figures, and widely used in elementary school in counting by ones and by groups, in the teaching of numeration, and in measuring volume.

<sup>16</sup> The root *ugol* "angle" in the word *ugolnik* has led to the erroneous translation of the latter as "protractor." Actually, as indicated by the illustrations in the previously cited work by Nikitin *et al.* (p. 81), the word refers either to a drawing-square or a drawing-triangle.

<sup>17</sup> As the word *ekher*, a loan-word from the French *équerre* "square," designates a special sort of square for outdoor work, I translate it as "surveying square." The instrument is depicted and described in the work cited in the previous note (pp. 93-94) and also in V. T. Snigiryev and Ya. F. Chekmarev, *Metodika arifmetiki. Posobiye dlya pedagogicheskikh uchilishch* (Moscow, 1948), pp. 118-19. It consists of two small boards (35×5×2.5 centimeters) fastened crosswise to each other in the shape of a Greek cross and mounted horizontally atop a stake about four feet in height. The instrument is used for staking out right angles by sighting along headless nails at the ends of the boards to the tops of sticks held vertically some distance away.

In the program for each class only new material has been pointed out. Along with the mastery of new material it is necessary to have systematic review of what has gone before. The teaching of each section of the course in arithmetic and also of the material of each quarter and each school year ends with review.

#### PROGRAM<sup>18</sup>

##### *Grade I<sup>19</sup>*

Counting up to 10; familiarity with numbers up to 10. Addition and subtraction within the limits of 10 (66 hours). (Here and elsewhere in each program the hours cited at each paragraph include the time in solving exercises and problems and in teaching metric measurement.)

Oral and written numeration up to 20. Addition and subtraction up to 20. Addition table. Increasing and decreasing a number by various units. Multiplication up to 20. Partitive division within the limits of 20 (100 hours).

Oral and written numeration up to 100. Addition and subtraction of round tens up to 100. Multiplication and division of round tens by a one-digit number within the limits of 100 (22 hours).

*Measures and exercises in measurement.* Meter, centimeter. Kilogram. Liter. Week, number of days in a week.

Familiarity with square, rectangle, triangle, and circle (their recognition and discrimination).

*Problems.* Solution of problems in one operation: by finding a sum or a difference, by finding the product (in the case of the repetition of a given number as an addend several times), by division into equal parts. Solution of problems in two operations.

*Review of the material covered* (10 hours).

##### *Grade II*

Review of the material covered in Grade I (12 hours).

Addition and subtraction up to 100. Comparison of numbers by subtraction (40 hours).

Multiplication and division up to 100; familiarity with measurement division; multiplication table and division with the aid of a table<sup>20</sup> (72 hours).

Increasing a number "so many" times; decreasing a number "so many" times; finding a fraction of a number; comparison of numbers

<sup>18</sup> The material in this section has already been published by Prof. Wirzup in the article cited in note 1. It has seemed useful to include my own translation nonetheless as it differs in a few points and is provided with explanatory notes.

<sup>19</sup> For a comparison of the contents of Russian and American first-grade arithmetic texts see my article, "Beginnings of Mathematical Education in Russia," *The Arithmetic Teacher*, VI, 1 (Feb., 1959), 6-11.

by division within the limit of 100 without the aid of tables (25 hours).

Oral and written numeration up to 1000. The four arithmetic operations on round hundreds up to 1000 with the use of oral methods of computation (16 hours).

*Measures and exercises in measurement.* Measures of length: kilometer, meter, centimeter. Measures of weight: kilogram, gram.

Measures of time: year, month, day, hour, minute (6 hours).

The straight line. Straight line segment and its measurement.

*Problems.* Solution of simple problems: by comparison by subtraction, by measurement division, by increasing and decreasing a number "so many" times, by finding a fraction of a number, by comparison by division.

Solution of composite problems in 2 to 3 operations.

*Review of material covered* (12 hours).

### Grade III

Review of what was covered in Grade II (12 hours).

The four operations on round tens and hundreds up to 1000 with the use of oral methods of computation.

Written computations within 1000: addition and subtraction of three-digit numbers; multiplication of two-digit and three-digit numbers by a one-digit number; division with remainder up to 100 with

<sup>20</sup> The author of the 5th-6th grade textbook cited in Note 3 states (p. 37) that Russian children are taught the use of multiplication and division tables "to speed up calculation," but it turns out that this involves more than the ordinary tables taught in our elementary schools. The student is informed that if he has to multiply, for example, 48 by 76, he should look either under 48 or 76 in a handbook containing appropriate tables. In one such handbook (P. N. Gorkin, *Tablitsy umnozheniya i delyeniya* [Moscow, 1955]) the 48 table for multiplication is given as follows:

	0	1	2	3	4	5	6	7	8	9	
<b>0</b>	0	4	9	14	19	24	28	33	38	43	<b>0</b>
<b>1</b>	48	52	57	62	67	72	76	81	86	91	<b>1</b>
<b>2</b>	96	100	105	110	115	120	124	129	134	139	<b>2</b>
<b>3</b>	144	148	153	158	163	168	172	177	182	187	<b>3</b>
<b>4</b>	192	196	201	206	211	216	220	225	230	235	<b>4</b>
<b>5</b>	240	244	249	254	259	264	268	273	278	283	<b>5</b>
<b>6</b>	288	292	297	302	307	312	316	321	326	331	<b>6</b>
<b>7</b>	336	340	345	350	355	360	364	369	374	379	<b>7</b>
<b>8</b>	384	388	393	398	403	408	412	417	422	427	<b>8</b>
<b>9</b>	432	436	441	446	451	456	460	465	470	475	<b>9</b>
	0	8	6	4	2	0	8	6	4	2	

Here the bold-face numbers on the left and right represent the multiplier's digit in the ten's place, and the bold-face numbers at the top represent the multiplier's digit in the one's place. To multiply 48 by 76, the student finds 364 at the intersection of the 7 row and the 6 column and annexes the number 8 at the bottom of this column, obtaining the answer 3648.

the use of tables; division of a three-digit number by a one-digit number (44 hours).

Oral and written numeration of large numbers up to a million. Addition and subtraction, multiplication and division of many-digit numbers with one-, two-, and three-digit numbers.

Addition and subtraction on the abacus.

Naming the components of arithmetic operations. Checking operations. Parentheses (simple cases) (98 hours).

*Measures and exercises in measurement.* Table of measures of length: kilometer, meter, decimeter, centimeter, millimeter. Table of measures of weight: ton, centner, kilogram, gram.

Table of measures of time: century, year, month, day, hour, minute, second (5 hours).

*Geometric material.* Measurement of segments. Simple measurements on location: staking out and measuring straight lines. Exercises in the development of visual estimation.

Rectangle and square; their sides and angles. Drawing a right angle, a square, and a rectangle with the help of a ruler and drawing-square and drawing-triangle (8 hours).

*Oral computations:* fluent computation within 100 and in round numbers within 1000. The use in oral computation of the rounding method and the commutative property of addition and multiplication.

*Problems.* Solution of simple arithmetic problems and composite problems in 2 to 5 operations in close connection with the study of the arithmetic operations.

Solution of problems involving the simple rule of three,<sup>21</sup> proportional division,<sup>22</sup> finding the unknown from two differences,<sup>23</sup> opposite motion<sup>24</sup> (19 hours). (Here are indicated only the hours for the solution of typical problems.)

*Review of material covered* (12 hours).

#### Grade IV

Review of the material covered in Grade III (12 hours).

Numeration of large numbers including millions and billions. Orders and periods. Addition and subtraction of many-digit numbers; the commutative property of addition; relationship between the

<sup>21</sup> The "rule of three" is extensively discussed in the teacher-training textbook by V. G. Chichigin, *Metodika prepodavaniya arifmetiki* (Moscow, 1952), 233 ff. An example of the "simple" rule of three is the problem: "2 kg of sugar cost 12 rubles. How much do 3 kg of sugar cost?" The "compound" rule of three is illustrated by the problem: "In 3 hours 5 pumps raised 1,800 buckets of water. How much water would 4 such pumps raise in 4 hours?" Chichigin suggests that the solution to such problems be taught by two methods, first by "reduction to a unit" and then with the help of proportion. For some historical notes on the "rule of three," a term which long ago disappeared from most textbooks of arithmetic, see the standard works by D. E. Smith *History of Mathematics*, and Vera Sanford, *A Short History of Mathematics*.

components of addition and subtraction; checking addition and subtraction.

Addition and subtraction on the abacus.

Multiplication and division of many-digit numbers; the commutative property of multiplication; relationship between the components of multiplication and division; checking multiplication and division; order of performing arithmetic operations (review) (44 hours).

*Compound denominate numbers.* Simple and compound denominate numbers. Reduction ascending and descending<sup>25</sup> of denominate numbers in the metric system of measurement. The four arithmetic operations on compound denominate numbers with the metric measures. Problems involving all operations with compound denominate numbers (26 hours).

*Geometric material.* Familiarity with area. Units of measurement of area. Measuring and computing the area of a rectangle and a square. Table of quadratic measures. The are and the hectare. Solving problems in computation of area. Construction on location of a right angle, a square, and a rectangle (14 hours).

Cubic measure. Familiarity with the cube: faces, edges, and vertices of a cube. The cube as a unit of measurement of volume. Meas-

<sup>25</sup> Under this heading a collection of problems and exercises for Grades V and VI includes the following: "Divide 765 into parts proportional to the numbers  $\frac{1}{2}$ ,  $\frac{1}{3}$ , and  $0.3$ ," "Divide 144 into 3 parts,  $x$ ,  $y$ , and  $z$ , so that  $x+y=3+4$  and  $y+z=4+5$ ." (S. A. Ponomaryov and N. I. Syrnev, *Sbornik zadach i uprashneny po arifmetike dlya 5-6 klassov semiletnei i srednei shkoly* [Moscow, 1958], pp. 187-188). Shebchenko (*op. cit.*, p. 190) gives the following rule for such problems: "In order to divide a number into parts proportional to given numbers, divide it by the sum of these numbers and successively multiply the resultant quotient by each of these numbers." [E.g. in order to divide 60 into parts proportional to 2, 3, and 5, divide it by 10 and successively multiply the resultant quotient 6 by 2, 3, and 5.]

<sup>26</sup> Under the heading "Problems in Finding the Unknown from Two Differences" a work on lesson plans in arithmetic for Grade III presents the following problem: "A state farm shipped 3 truckloads of potatoes to the city the first day, and 5 truckloads the second. On the second day 6 metric tons more were shipped than on first day. How many tons of potatoes were there in one truckload?" The solution is formulated as follows:

1. How many more truckloads of potatoes were shipped on the second day than on the first day?

$$5 \text{ truckloads} - 3 \text{ truckloads} = 2 \text{ truckloads}$$

2. How many tons of potatoes were there in one truckload?

$$6 \text{ tons} \div 2 = 3 \text{ tons}$$

(V. A. Ignatyev, N. I. Ignatyev, and Ya. A. Shor, *Plany uchebov po arifmetike dlya 3 klassa nachalnoi shkoly* [Moscow, 1958], p. 92.)

<sup>27</sup> Under the heading "Solution of Problems in Opposite Motion" the work just cited presents (pp. 161-62) the following "model problem": "The distance between two opposite walls is 6 meters. Two students, one of whom goes 45 centimeters (the length of his step) in one second and the other 55 centimeters, start simultaneously from opposite walls toward each other. After how many seconds do they meet?" The solution is formulated as follows:

1. In one second the two students draw how many centimeters closer to each other?

$$45 \text{ cm} + 55 \text{ cm} = 100 \text{ cm} = 1 \text{ m}$$

2. After how many seconds do they meet?

$$6 \text{ m} \div 1 \text{ m} = 6 \text{ (sec.)}$$

<sup>28</sup> Reduction ascending: 1,235 centimeters = 12 meters 35 centimeters. Reduction descending: 12 meters 35 centimeters = 1,235 centimeters.



uring and computing the volume of right-angular bodies (boxes, chests, rooms). Table of cubic measures. Solving problems involving computation of volume (14 hours).

Measures of time. Table of measures of time (review); reducing measures to higher and larger order. The four operations on compound denominate numbers with measures of time (simple cases).

Problems involving computation of time within the limits of a day, year, and century (the last in whole years) (26 hours).

*Simple fractions:*  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{1}{5}$ ,  $\frac{1}{10}$ . Partition. Numerator and denominator of fractions. Reduction of fractions. Addition and subtraction of fractions whose denominators are alike or multiples of each other. Solution of problems in finding various parts of a number (18 hours).

*Oral computations.* Fluent computation within the limits of 100 and with round numbers within 1000. Use in simple cases of successive multiplication and division (by 2, 4, 8, etc.). Short multiplication by 5, 50, 25.

*Problems.* Solution of composite arithmetic problems in 2 to 6 operations in connection with studying the arithmetic operations.

Problems in computing the arithmetic mean. Problems solved by the method of ratios. Problems in finding two numbers from the sum and ratio<sup>26</sup> (15 hours). (Here are indicated only the hours for the solution of typical problems.)

Review of material covered (29 hours).

<sup>26</sup> This designation covers also the finding of more than two numbers by the method indicated, as in the following example: "A tourist travelled 2,200 kilometers, going twice as far by boat as by horse, and four times as far by railway as by boat. How many kilometers did the tourist travel by boat, by horse, and by railway?" The indicated solution utilizes the "method of parts" whereby the student finds the sum of the parts (1+2+8), the value of one part (2,200÷11), and the value of the remaining parts (200×2 and 200×8). (V. T. Snigiryev and Ya. F. Chekmayev, *op. cit.*, pp. 75-76.)

## ASTRONOMERS USE ROCKETS TO PROBE SUN'S BEHAVIOR

Rockets soaring high into earth's atmosphere, instruments that make man-made solar eclipses, and pencil and paper to interpret the information so gained are helping astronomers learn about the sun's behavior.

At the American Astronomical Society meeting, scientists reported results of their instrumental and theoretical probings of the star nearest earth, the sun. It was suggested that solar prominences are not uniform clouds of luminous gas, as many have thought.

Observations with the solar coronagraph, a device to view the sun as if it were in eclipse, "clearly show" that prominences possess a string-like structure, sometimes appearing like tangled skeins of thread. In prominences associated with sunspots, the filaments assume distinctive forms of which loops are the simplest and most characteristic, and the filaments themselves show internal structure.

Photographs of the sun taken in the far ultraviolet or solar Lyman alpha showed that there is hydrogen in the space between the earth and the sun. The rocket flights during which the sun was examined in the Lyman alpha hydrogen line were made July 21, 1959.

## Effect of Dogmatism on Critical Thinking\*

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We live in an unpredictable age. It was possible in the closed society of the Middle Ages to foresee both the problem and the most appropriate solution. In the middle of the 20th century we can no longer perceive the particular problems which will be paramount in the next five or ten years.

This situation calls for a new approach. Emphasis must be placed upon the development of generalized ways of attacking problems and on knowledge which can be applied to a wide range of new situations. It is now impossible for the individual to attain all the necessary knowledge for each and every new situation. Nevertheless he must act and even in the situation which has a minimal number of unique characteristics he must be able to successfully analyze the problem and evaluate the knowledge and/or skills for its solution.

Although the need for critical thinking is recognized, there is wide divergence in what is meant by the term. One group of educators concludes that it has at least reference to five abilities: (1) the ability to define a problem (2) the ability to select pertinent information for the solution of a problem (3) the ability to recognize stated and unstated assumptions (4) the ability to formulate and select relevant and promising hypotheses, and (5) the ability to draw conclusions validly and to judge the validity of inferences.<sup>1</sup> A group concerned with improvement of critical thinking suggests that more attention be given to helping students "develop problem-solving methods which will yield more complete and adequate solutions in a wide range of problem situations."<sup>2</sup> This suggestion follows their observation that when confronted with problems individuals in general behave as follows:

1. They tend to avoid real problem-solving.
2. They apply only a limited stock of techniques to solve them.
3. They are satisfied with a partial solution.
4. They change the problem completely.
5. They escape from it entirely.<sup>3</sup>

These behaviors indicate the influence of emotional factors on critical thinking. This relationship between personality and cognitive variables was first established by the research of Else Frenkel-

\* Paper presented at the annual meeting of the American Association for the Advancement of Science, 1959 in Chicago, Illinois.

<sup>1</sup> Paul L. Dressel and Lewis B. Mayhew, *General Education: Explorations in Evaluation*, Washington, D. C. American Council on Education, 1954, 179-181.

<sup>2</sup> Benjamin S. Bloom, Editor, *Taxonomy of Educational Objectives*, New York, Longmans, Green and Co., 1956, 43.

<sup>3</sup> *Ibid.*, 42, 43.

Brunswik<sup>4</sup> who found that as a result of the early parent-child relationships there emerge degrees of variance in the ability of the youth to tolerate ambiguity and that this emotional and social ambivalence manifests itself in the cognitive spheres (thinking, perception, and memory). Postman and his associates<sup>5</sup> concluded from their research that the individual sets up a perceptual defense against inimical stimuli. Anderson<sup>6</sup> from experience in working with the Stanford Binet Intelligence Test has provided examples of this perceptual defense. Allport<sup>7</sup> observed in his study of rumor that what leads to obliteration of some details and falsification of others is that the force of the intellectual and emotional context existing in the individual's mind leads to the assimilation of ideas in accordance with the values resident within the individual. This, Maslow<sup>8</sup> concludes, wards off threatening aspects of reality which at the same time provides the individual with a compensatory feeling that he understands it. This form of thinking is referred to as dogmatic.

Such behaviors as those noted above lower the efficiency of the individual in critical thinking. They may be minimized through instruction and experience in problem-solving methods, but not eliminated. When we examine them we note that emotional effects are exerting a pervasive influence on the outcome. These individuals apparently try to cope with a situation through the use of distortion, narrowing or withdrawal. They do not tolerate ambiguity and move toward "closure" without sufficient consideration of the various aspects necessary for the solving of the problem.

Of course others confront new experience very differently. These approach it in all its detail. They analyze, evaluate, discard or integrate part or all of it. The more open-minded they are, the more perceptively they examine different aspects of the experience, try to clarify the ambiguity, and strive to see the relationship among parts.

Solomon<sup>9</sup> found in the use of the scientific method among college students that the open-minded showed greater ability to discard preconceived ideas and to integrate or accept new and scientifically demonstrated facts. These open-minded persons in whom there is almost a complete absence of defenses and an increase in spontaneity

<sup>4</sup> Else Frenkel-Brunswik, "Intolerance of Ambiguity as an Emotional Perceptual Personality Variable," *Journal of Personality*, 1949, 18, 108-143.

<sup>5</sup> L. Postman, J. Bruner, and E. McGinis, "Personal Values as Selective Factors in Perception," *Journal of Abnormal and Social Psychology*, 1948, 43, 142-154.

<sup>6</sup> Gladys L. Anderson, "Qualitative Aspects of the Stanford-Binet" in *An Introduction to Projective Techniques*, Harold H. and Gladys L. Anderson, Editors, New York, Prentice-Hall, Inc., 1951, 581-605.

<sup>7</sup> Gordon W. Allport and Leo Postman, *The Psychology of Rumor*, New York, Henry Holt and Company, 1947, Chapter 6.

<sup>8</sup> A. H. Maslow, *Motivation and Personality*. New York: Harper and Bros., 1954.

<sup>9</sup> Marvin D. Solomon, *The Personality Factor of Rigidity as an Element in the Teaching of the Scientific Method*. Unpublished Doctoral Thesis, Michigan State College, 1953.

and honesty resemble the self-actualizing individuals described by Maslow.<sup>10</sup>

The distinct difference in the approach to critical thinking between the open and closed mind led to the assumption that in situations requiring the performance of the higher thought processes the low dogmatic individual would be more efficient than the high.

#### PURPOSE OF THE STUDY

The purpose of the study was to compare those who were low with those who were high in dogmatism with reference to their ability in critical thinking as indicated in problem-solving.

#### HYPOTHESIS

It was hypothesized that those who were open-minded or low in dogmatism would solve correctly more critical thinking problems than those who were high in dogmatism.

#### SAMPLE

Male and female freshmen students from Olivet College, Alma College, and Michigan State University in Michigan, and Salem College, West Virginia participated in the study. This gave a grand total of 500 students.

#### PROCEDURES

The Dogmatism Scale Form E developed and standardized by Milton Rokeach<sup>11</sup> was used as the means of classifying the students in dogmatism.

One hundred problems in critical thinking involving analysis and evaluation were administered to a random sample of 100 students and the most discriminating 50 items used in the study. The final items were chosen from the following forms:

Taxonomy of Education Objectives—items 1-10, 13-15, 26-36.

A Test of Problem-Solving, High School Edition, Form A. 11, 12, 16-18, 37-41, 43-47.

A Test of Critical Thinking, Form G 19-25, 42, 48-50.

Each student participating in the study was administered the Dogmatism Scale Form E, and the critical thinking problems. The 150 students with the highest scores in dogmatism and 150 with the lowest were arbitrarily selected for the comparison of results in critical thinking. Two of the most discriminating items, 19 and 44 (Appendix) which yielded the greatest difference in performance between

<sup>10</sup> *Ibid.*, Maslow.

<sup>11</sup> Milton Rokeach. *The Open-Closed Mind*. New York, Basic Books, 1960, Chapter 2.

the low and high dogmatic groups were selected for study. A careful look at these two items reveals these characteristics:

1. They require the consideration of a number of factors for their solution.
2. They are closely related to security needs, e.g. health insurance.
3. They provide a means for choosing one or other pole of a dichotomy e.g. who should run America (government or the people).
4. They provide a choice for the individual who desires to narrow, distort or escape the problem e.g. response 5 in both items.

#### ANALYSIS OF THE DATA

Those low in dogmatism were superior in critical thinking to those high in dogmatism. The "t" test gave a significant difference at the one per cent level (Table 1).

TABLE 1  
A COMPARISON OF THE NUMBER OF CORRECT RESPONSES TO PROBLEMS IN  
CRITICAL THINKING BETWEEN LOW AND HIGH DOGMATIC GROUPS  
N 300

Group	Means	Diff/M	t	P
Low	26.91			
High	23.21	3.70	2.81	.01
		$\alpha.01 = 2.59$		

A comparison was made of the responses of the low and high dogmatic groups to problems number 19 and 44. With reference to number 19, the Chi Square gave a significant difference above the one per cent level (Table 2). This table shows that 35 per cent of the low

TABLE 2  
A COMPARISON OF THE RESPONSES OF THE LOW AND HIGH GROUPS ON ITEM 19  
N 300

Group	Responses				
	1	2	3	4	5
Low	75	8	7	8	52
High	37	15	7	23	68

Chi Square = 21.09

Significance at the one per cent level = 13.28

The correct response is Number 1

and 46 per cent of the high chose assumption number five,—"some luxuries are habit forming." The correct response is number one, "luxuries are never good for you," but we note that this response condemns luxuries and in our culture they are symptomatic of success and status. It is reasonable to assume that these students nar-

rowed, or distorted the real nature of the problem and responded in accordance with their preformed value system.

With reference to problem number 44, the Chi Square gave a significant difference above the one per cent level (Table 3). Twice as

TABLE 3  
A COMPARISON OF THE RESPONSES OF THE LOW AND HIGH GROUPS ON ITEM 44  
N 300

Group	Responses				
	1	2	3	4	5
Low	15	52	23	7	53
High	15	7		23	105

Chi Square = 47.16

Significance at the one per cent level = 13.28

The correct response is number 2.

many of the high chose response five. This may be the result of an early "closure," a decision reached without a full consideration of all factors involved, apparently a failure to recognize that the decision as to whether a national health insurance program was needed was more basic and should be given precedence over the question of whether it would provide greater security.

#### CONCLUSIONS

The low dogmatics are more successful than the high in critical thinking. The high dogmatics have the greater percentage of errors in those problems which require the study of several factors or criteria for decision and the deferring of a conclusion until each factor has been judiciously considered. Apparently the high dogmatic has difficulty in tolerating ambiguities and is thus impelled toward a "closure" before full consideration is given to each piece of contributing evidence. This sometimes results in the perceptual distortion of facts and in a conclusion which does not encompass all elements of the problem. This may be expected when a narrowing or distortion of parts of the complete problem can change it from one which threatens to one which offers gratification of security needs. Occasionally the significance of parts or of the whole problem is ignored and a solution attained which fits harmoniously into the preformed value pattern.

#### IMPLICATIONS

Improved teaching methods which provide greater opportunity for students to engage in critical thinking should be encouraged. This however is not enough. Educators need to become aware that the



poor performance of capable individuals may be due to closed minds or dogmatism. Since dogmatism adversely affects the quality of thought, attention should be given to the development of the understanding and skill necessary for coping with the problem.

The present indications are that in interpersonal relationships in which there is a permissive safety the dogmatic individual more easily lowers his defenses and progress is toward perceiving problems as they really exist.

#### APPENDIX

19. This question is an argument which involves an unstated assumption. Select from the list the assumption which is left unstated.

##### *Assumptions*

1. Luxuries are never good for you.
  2. Some habit-forming things are good for you.
  3. Habit-forming things are never good for you.
  4. Some things which are good for you are luxuries.
  5. Some luxuries are habit-forming.
- ( ) You must agree that some habit-forming things are not good for you, for you admit some of them are luxuries.
44. This question is based on the following statement, which is quoted from an advertisement which appeared several years ago. At that time a national health insurance plan had been proposed by the Federal Security Agency and was supported by the President.

**"Who runs America? The Congress? The President? or YOU AND THE MAN NEXT DOOR?"**

Running America is the joint job of 150,000,000 people. It's the biggest job in the world today—keeping it running for liberty and freedom. And the whole world's watching to see whether Americans can do it.

In much of the world today, the people have resigned from running their own countries. Others have been quick to step in—first with promises of security—and then with whips and guns—to run things their way. The evidence is on every front page in the world, every day.

Freedom comes under attack. The reality of war has made every American think hard about the things he's willing to work and fight for—and freedom leads the list. But that freedom has been attacked here recently—just as it has been attacked in other parts of the world. One of the most serious threats to individual freedom is the threat of Government-dominated Compulsory Health Insurance, falsely presented as a new guarantee of health "security" for everybody.

A reader is trying to decide whether he should support this purpose of the writer. His judgment should depend, fundamentally, on the question of:

1. Who should run America.
2. Whether the American people really need a national health insurance program.
3. Whether running America is the biggest job in the world.
4. Whether freedom is under attack in much of the world.
5. Whether it is true that *everybody* would gain greater security from a health insurance program.

## An Apparatus for Producing a Non-Luminous Candle Flame

Harold J. Abrahams, Stephen Rosenzweig,\*  
and Howard Friedman\*

*Public High Schools of Philadelphia, Pennsylvania*

It is not often enough that a member of a class asks a question which a teacher may answer by suggesting that the questioner set up an experiment resulting in a clean-cut and convincing answer.

In the study of oxygen and combustion, teachers discuss the beautiful phenomenon of flame, its cause, and the reason for luminosity or its lack. Some time during the discussion, the definition of a flame as "a glowing or incandescent vapor" is either given or developed. The distinction between luminous and nonluminous gas flames is pointed out and explained on the basis of air supply. Emphasis is laid upon the fact that if the air supply is adequate, all or nearly all of the fuel is burned and the products escape as gases, but if the air supply is not adequate, some of the carbon escapes conversion into carbon dioxide gas, and, instead, forms particles of solid carbon which, due to the high temperature of the flame, become hot enough to glow or incandesce, thus producing luminosity. The analogy between the luminous gas flame and that of a candle is discussed, and it is explained that both flames owe their luminosity to the presence of glowing, unburned carbon particles, whose existence can be proved by holding a cold white surface in each of the two flames.

At this point the question perennially raised by a member of the class is, "if you could somehow get a supply of air into the candle flame, as can be done in the case of the gas flame, would it, too, produce a non-luminous flame, such as is produced by the Bunsen burner, and for the same reason?"

Having heard this question asked in class after class, for many years, it seemed high time that an attempt was made to put together a device which would answer the question with an eloquence unattainable verbally.

The boys who volunteered to set up this device were assigned the work of drilling holes in household candles from top to bottom, close to the wicks. A few candles suitably perforated for the intake of air were produced. Blowing the breath into these holes through glass tubes, while the candles were burning, produced the desired result—a Bunsen or non-luminous flame. However, a design for a better apparatus came to mind. This was assembled, tried, and found satisfactory. The method of its construction follows:

\* Members of the Chemistry class.

Two blocks of paraffin wax, three inches wide, four inches high, and one-half or three-quarters of an inch thick are warmed, each on one side, until they are soft and about to start melting. Then the two softened sides are brought together and held in position until permanently fastened together. Two more such blocks are similarly fastened together so that there are now two thick blocks, each consisting of two thinner ones. Now a brass blowpipe is heated gently and pressed against a side of one of the newly made thick blocks until it melts the wax and sinks into it to a depth of about one-half of the thickness of the blowpipe. In selecting the exact location for this embedding operation, it should be remembered that the tip of the blowpipe should protrude about three-eighths of an inch above the top of the block of wax, and be centered with respect to the left and right sides of the latter. When the blowpipe is in position a string or cord is also put into position from one-half inch above the top to one-half inch below the bottom of the block in the manner of a candle wick (see "A" in illustration). Then the second thick block of paraffin is warmed gently on one side and, when soft enough, pressed against the other block, making a "sandwich" of the blowpipe and string. Now, a one inch length of hygrometer- ("wet bulb thermometer") wick, stretched over a one inch length of glass "connection tubing" is passed over the top of the blowpipe (see "B" in illustration). The string wick, trimmed down to about one-quarter of an inch above the top of the wax, is kindled, causing some paraffin wax to melt and be drawn up into the hygrometer-wick. After this, the hygrometer-wick is kindled, and the flame of the string wick is extinguished by "pinching" it between wet fingers or forceps. An aspirator bulb is then attached to the other end of the blowpipe and air is gently sent through

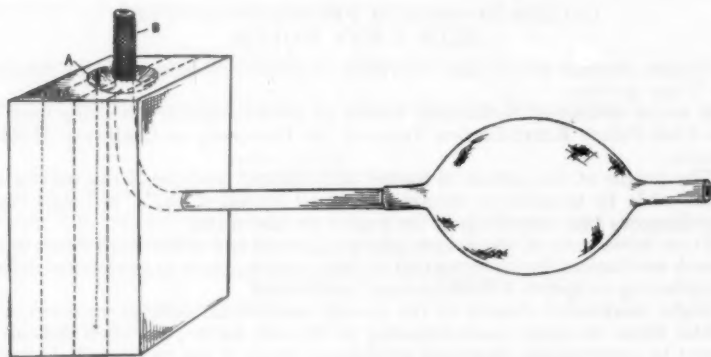


FIG. 1. An apparatus for producing a non-luminous candle flame.

blowpipe and flame of the burning paraffin wax. After a few attempts, the proper technique of aspirating the flame is quickly learned. If the frequency of compression of the bulb is correct, a non-luminous (Bunsen) flame will result.

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#### LETTER TO THE EDITOR

Readers of SCHOOL SCIENCE AND MATHEMATICS may be interested in the following letter to the Editor:

"Mr. George G. Mallinson  
Editor, SCHOOL SCIENCE AND MATHEMATICS  
Western Michigan University  
Kalamazoo, Michigan

Dear Mr. Mallinson:

It was nice to see an old favorite appear in your pages. I am referring to Mr. Amir-Moéz's solution to the angle bisectors theorem.

Perhaps your readers would like to be reminded that a very complete article on this problem (which is also known as the Lehmus-Steiner theorem) appeared in the pages of the SCHOOL SCIENCE AND MATHEMATICS back in June 1939. The article was written by Mr. David McKay and it contains several geometric proofs—both direct and indirect—algebraic proofs, and trigonometric proofs.

Very sincerely yours,

JULIUS H. HLAVATY, *Director*  
Mathematics Commission Program  
Commission on Mathematics  
425 W. 117th Street  
New York 27, New York

JHH:gh

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#### CAUSES OF SPEECH PROBLEMS SPOTTED WITH X-RAY MOVIES

Hidden physical defects that contribute to speech problems are being detected by X-ray movies.

A movie technique, technically known as cinefluorography, is being used by the Cleft Palate Rehabilitation Team at the University of California Medical Center.

The tongue of the patient is coated with barium, and the X-ray movies are made while he pronounces certain key vowel sounds—"ah," "ee," and "oo." Simultaneous tape recordings of the sounds are also made.

From movements of the tongue, pharynx, uvula and other components of the speech mechanism during formation of these sounds, clues to anatomical defects contributing to speech difficulties may be obtained.

Slight anatomical defects of the speech mechanism, such as excessive adenoidal tissue or subtle malfunctioning of the soft palate, are often difficult to detect by conventional diagnostic techniques. Study of the movies will, however, often reveal such defects.

## PROBLEM DEPARTMENT

Conducted by Margaret F. Willerding

San Diego State College, San Diego, Calif.

*This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.*

*All readers are invited to propose problems and to solve problems here proposed. Drawings to illustrate the problems should be well done in India ink. Problems and solutions will be credited to their authors. Each solution or proposed problem sent the Editor should have the author's name introducing the problem or solution as on the following pages.*

*The editor of the Department desires to serve her readers by making it interesting and helpful to them. Address suggestions and problems to Margaret F. Willerding, San Diego State College, San Diego, Calif.*

### SOLUTIONS AND PROBLEMS

Note. Persons sending in solutions and submitting problems for solutions should observe the following instructions.

1. Solutions should be in typed form, double spaced.
2. Drawings in India ink should be on a separate page from the solution.
3. Give the solution to the problem which you propose if you have one and also the source and any known references to it.
4. In general when several solutions are correct, the one submitted in the best form will be used.

### LATE SOLUTIONS

2692. Lowell Van Tassel, San Diego, Calif.

2695. J. Byers King, Denton, Md.

2696. H. J. Fletcher, Provo, Utah

2701. Proposed by C. W. Trigg, Los Angeles, Calif.

The consecutive odd numbers are grouped as follows: 1, (3, 5); (7, 9, 11); (13, 15, 17, 19); . . . Find the sum of the numbers in the  $n$ th group.

*Solution by John F. Graney, Culver, Ind.*

It is clear that these are  $n$  odd numbers in the  $n$ th group. If  $T$  = the total number of odd numbers which precede the  $n$ th group then

$$T = 1 + 2 + 3 + \cdots + (n-1) = \frac{n(n-1)}{2}.$$

Since the  $n$ th odd number is  $2n-1$  it follows that the first number in the  $n$ th group is

$$2 \left[ \frac{n(n-1)+2}{2} \right] - 1 = n(n-1) + 1.$$

Using this result and the fact that the sum of the first  $n$  odd integers  $= n^2$ , the sum,  $S$ , of the  $n$ th group is readily obtained.

$$S = n(n-1) + 1 + n(n-1) + 3 + \cdots + n(n-1) + 2n-1$$

$$S = n^2(n-1) + n^2$$

$$S = n^3$$

Solutions were also offered by Lloyd G. Bishop, Leaside Ontario, Canada; W. G. Gingery, Bloomington, Ind.; Robert Guderjohn, Eugene, Ore.; Louis J.

Hall, Bloomington, Ind.; Herbert R. Leifer, Pittsburgh, Pa.; Bernard T. Pleimann, Gardena, Calif.; W. R. Talbot, Jefferson City, Mo.; and Dale Woods, Kirksville, Mo.

**2702.** *Proposed by Cecil B. Read, University of Wichita, Wichita, Kans.*

If the smallest prime factor of a number is greater than the cube root of that number, show that the remaining factor is also prime.

*Solution by W. R. Talbot, Jefferson City, Mo.*

Let  $N = a^3 = pq$  where  $p$  is the smallest prime factor of  $N$  and exceeds  $a$ . We exclude as non-pertinent the case of  $q = 1$ . Let  $q > 1$  be composite, say  $q = rs$ . We must have  $r > p$  and  $s > p$ . Then  $a^3 = N = prs > p^3 > a^3$  which is impossible. Then if  $q > 1$ ,  $q$  cannot be composite.

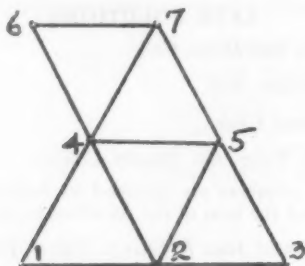
Solutions were also offered by W. G. Gingery, Bloomington, Ind.; John F. Graney, Culver, Ind.; Louis J. Hall, Bloomington, Ind.; Dale Woods, Kirksville, Mo.; and the proposer.

**2703.** *Proposed by Howard Grossman, New York, N. Y.*

If all the points of the plane are divided into two sets in any way, at least one of the sets must contain the vertices of an equilateral triangle.

*Solution by Walter R. Talbot, Jefferson City, Mo.*

Since equilateral triangles may be drawn in the plane, the problem is to show that the assumption of two vertices being in one set and the third in the other must lead to an impossibility.



Assume a pattern of equilateral triangles as in the figure with vertices belonging to set  $A$  or set  $B$ . Assume, for example, that 4 and 7 are in  $A$ . Then 5 and 6 are in  $B$ . For triangle 156, 1 must be  $A$ . Then 2 is  $B$ . If 3 is  $B$ , triangle 253 is all  $B$ . If 3 is  $A$ , triangle 137 is all  $A$ . In either case the assumption that two vertices must lie in one set and the third in the other breaks down and the three vertices lie in one set.

A solution was also offered by the proposer.

**2704.** This is the same problem as 2667 which appeared in the March 1959 issue of *SCHOOL SCIENCE AND MATHEMATICS*. Its solution appears in the October 1959 issue.

**2705.** *Proposed by Brother Felix John, Philadelphia, Pa.*

Solve the equation:

$$(n^2 - 5n - 24)^{1/2} + (n^2 + 5n - 24)^{1/2} = (n^2 + 10n + 24)^{1/2} - (n^2 - 10n + 24)^{1/2}$$

*Solution by Herbert R. Leifer, Pittsburgh, Pa.*



Set

$$-2[(n^2-5n-24)-(n^2+5n-24)] = (n^2+10n+24) - (n^2-10n+24).$$

Dividing by the given equation, we get

$$-2[(n^2-5n-24)^{1/2} - (n^2+5n-24)^{1/2}] = (n^2+10n+24)^{1/2} + (n^2-10n+24)^{1/2},$$

adding the given equation, we get

$$-(n^2-5n-24)^{1/2} + 3(n^2+5n-24)^{1/2} = 2(n^2+10n+24)^{1/2}.$$

This is readily solved by squaring to give

$$n = (2560/39)^{1/2}.$$

Solutions were also offered by J. W. Lindsey, Amarillo, Texas; W. R. Talbot, Jefferson City, Mo.; and the proposer.

**2706. Taken from Mathematical Pie.**

How many different digits are need by a builder in order to number all of 288 houses in a street? (front doors only.)

*Solution by Walter Talbot, Jefferson City, Mo.*

If there had been 300 houses placed 0 through 299, the units place would take 30 of each digit; the tens place would take 30 of each digit except 0 for which there would be 20; and the hundreds place would require 100 of each of 1 and 2. For the houses numbered 0 and 289 through 299, we deduct 2 each of 0 and 8; 12 each of 2 and 9; 1 each of 3, 4, 5, 6, 7 leaving 159 of 1; 148 of 2; 59 each of 3, 4, 5, 6, 7; 58 of 8; and 48 each of 9 and 0 for a grand total of 756 numerals.

A solution was also offered by Dale Woods, Kirksville, Mo.

**HIGH SCHOOL HONOR ROLL**

The Editor will be very happy to make special mention of high school classes, clubs, or individual students who offer solutions to problems submitted in this department. Teachers are urged to report to the Editor such solutions.

**Editor's Note:** For a time each high school contributor will receive a copy of the magazine in which the student's name appears.

For this issue the Honor Roll appears below.

2701, 2702, 2704, 2705, 2706. *Lee Mitchell, Ann Arbor, Mich.*

2701. *Ray Long, Bastrop High School, Bastrop, Texas.*

2701. *Floyd D. Wilder, Bethany Nazarene College, Bethany, Okla.*

2701. *Charles S. Hertz, Jr., Phillips Exeter Academy, N. H.*

**PROBLEMS FOR SOLUTION**

2725. *Proposed by Louis J. Hall, Bloomington, Ind.*

There is an old problem in which a person beginning employment is given a choice of a pay schedule of starting pay plus a raise of either \$150 a year or \$50 every six months, the raises to take effect at the end of each respective period of employment. It can be verified that the \$50 every six months is the better choice. This is also true if the choices are \$200 and \$50 respectively, but not true if the choices are \$250 and \$50. Find an expression for a choice of "B" dollars raise per year of "A" dollars every six months which will be equivalent in total earnings in  $n$  years for  $n$  greater than one.

2726. *Proposed by Lowell Van Tassel, San Diego, Calif.*

A neurologist wishes to calculate the total number of different ways that nerve

cells can be linked pair wise, that is, two-by-two. For a small animal brain of only a million cells, he calculates.

$$10^{2,763,000}.$$

(a) Comment of his answer; (b) find a general solution for a brain of  $n$  ( $n$  even and very large) cells.

**2727.** Proposed by W. R. Talbot, Jefferson City, Mo.

Without use of decimals determine whether the quantity

$$13\sqrt{5}-9\sqrt{10}+7\sqrt{17}-5\sqrt{34}$$

is positive or negative.

**2728.** Proposed by Cecil B. Read, Wichita, Kans.

Find a two digit number equal to the square of the ten's digit plus the square of the sum of its digits.

**2729.** Proposed by G. P. Speck, Virginia, Minn.

Prove or disprove the following conjecture: Given any real number  $r$ , for every  $\epsilon > 0$  there exists a pair of integers  $(m, n)$  such that  $|m - nr| < \epsilon$ .

**2730.** Proposed by Felix John, Philadelphia, Pa.

Show that every prime factor is contained in  $(n+r)!$  as often at least as it is contained in  $n!$ .

For the next four months this department will offer two problems especially for high school students. All teachers are encouraged to have their students submit problems and solutions for this special section of the Problem Department.

Students sending in solutions and submitting problems for solution should observe the following instructions:

1. Each solution must be in typed form, double spaced.
2. Drawings, in India Ink, should be on a separate page from the solution or proposal.
3. Problems and solutions should be in the same form that appears in the Journal.
4. Each problem should be on a separate sheet of paper.
5. In general when several solutions are correct, the one submitted in the best form will be used.

#### STUDENT PROBLEMS FOR SOLUTION

**S-3.** Proposed by Lowell Van Tassel, San Diego, Calif.

Calculate, in the duodecimal system (radix = 12,  $10 = t$ ,  $11 = e$ ) the following values, retaining results in the duodecimal system. (Remark: It is considered cheating to multiply the translated problems in the decimal system and then re-convert.)

- |                           |                                      |
|---------------------------|--------------------------------------|
| (a) (tete) <sup>2</sup>   | (d) (toe) <sup>5</sup>               |
| (b) (ete) <sup>2</sup>    | (e) $\frac{\text{toot}}{\text{tet}}$ |
| (c) (tete) $\times$ (ete) |                                      |

**S-4.** Taken from Mathematical Pi.

The Geometry teacher, feeling the need for some fresh air, glanced at the clock as he went out at some time between 4 and 5 P.M.

On his return about 3 hours later, he noticed that the hands of the clock had exactly changed places. What was the time of his departure?

## Books and Teaching Aids Received

SINGER ELEMENTARY SCIENCE SERIES, by George Willard Frasier, Helen Dolman MacCracken and Donald Gilmore Decker. All Cloth. All 16×22 cm. 1959. The L. M. Singer Company, Inc., Syracuse, New York.

Book 1—Science for You, 160 pages.

Book 2—Science All the Year, 192 pages.

Book 3—Science Adventures, 224 pages.

Book 4—Science Discoveries, 288 pages.

Book 5—Science Experiments, 320 pages.

Book 6—Science Problems, 352 pages.

Teacher's Guides for the above books. All paper. 21×28 cm.

Book 1—108 pages.

Book 2—122 pages.

Book 3—140 pages.

Book 4—169 pages.

Book 5—190 pages.

Book 6—206 pages.

GAMES FOR LEARNING MATHEMATICS, by Donovan A. Johnson. Paper. 176 Pages. 20×27 cm. 1960. J. Weston Walch, Box 1075, Portland, Maine. Price \$2.50.

YES, MATH. CAN BE!, Teacher Edition, by Louis Grant Brandes. Paper. Pages iv+263. 20×27.5 cm. 1960. J. Weston Walch, Box 1075, Portland, Maine.

THE EDUCATION OF TEACHERS: CURRICULUM PROGRAMS, Report of the Kansas TEPS Conference, 1959. Cloth. ix+453 Pages. 14.5×23 cm. 1959. National Commission on Teacher Education and Professional Standards, National Education Association of the United States, 1201 Sixteenth, Street, Northwest, Washington 6, D. C. Price \$3.50.

TEACHERS OF CHILDREN WHO ARE HARD OF HEARING, by Romaine P. Mackie, Chief, Exceptional Children and Youth and Don A. Harrington, Specialist, Speech and Hearing, Exceptional Children and Youth. Paper. Pages ix+70. 15×23 cm. 1959. Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. Price \$.35.

UNITIZED EXPERIMENTS IN ORGANIC CHEMISTRY, by Ray Q. Brewster, Calvin A. Vanderwerf, and William E. McEwen, Professors of Chemistry, University of Kansas. Paper. Pages xii+200. 20.5×28 cm. 1960. D. Van Nostrand Company, Inc., 257 Fourth Avenue, New York 10, New York.

CRIBICLES: THE STORY OF CHEMISTRY, From Ancient Alchemy to Nuclear Fission, by Bernard Jaffe. Paper. 240 Pages, 10.5×18 cm. 1960. Fawcett World Library, 67 West 44th Street, New York 36, N. Y. Price \$.50.

REAL VARIABLES, by John M. H. Olmsted. Cloth. Pages xvi+621. 15×23.5 cm. 1959. Appleton-Century-Crofts, Inc., 35 West 32nd Street, New York 1, N. Y. Price \$9.00.

ANALYTIC GEOMETRY AND AN INTRODUCTION TO CALCULUS, by A. Clyde Schock and Bernard S. Warshaw. Cloth. 165 Pages. 15×23 cm. 1960. Prentice-Hall, Inc. Englewood Cliffs, N. J. Price \$3.96.

FINANCING HIGHER EDUCATION, 1960-70, by Dexter M. Keezer. Paper. Pages vii+304. 15×23 cm. 1959. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 36, New York. Price \$2.00.

STATISTICS OF BONDS SOLD FOR PUBLIC SCHOOL PURPOSES, October 1953-June 1959, by Stanley V. Smith and E. Joan McMurray, Analytical Statisticians. Paper. Pages iv+15. 20×26 cm. 1959. Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. Price \$.20.

PHYSICS QUESTIONS AND PROBLEMS WITH ANSWERS, by Alexander Efron, author of BASIC PHYSICS. Paper. 52 Pages. 21.5×28 cm. 1959. John F. Rider Publisher, Inc., 116 West 14th Street, New York 11, N. Y. Price \$1.50.

A BIRD IS BORN, E. Bosiger and J. M. Guilcher. Cloth. 92 Pages. 13.5×19.5 cm. 1960. Sterling Publishing Company, Inc., 419 Fourth Avenue, New York 16, N. Y. Price \$2.50.

A BUTTERFLY IS BORN, by J. P. Vanden Eckhoudt. Cloth. 90 Pages. 13.5×19.5 cm. 1960. Sterling Publishing Company, Inc., 419 Fourth Avenue, New York 16, N. Y. Price \$2.50.

THE SKY IS OUR WINDOW, by Terry Maloney. Cloth. 128 Pages. 16×25.5 cm. 1960. Sterling Publishing Co., Inc., 419 Fourth Avenue, New York 16, N. Y. Price \$3.95.

ADVENTURE WITH WEATHER, A Capitol Adventure Kit with 96-page illustrated handbook, by Harry Milgrom, Supervisor of Science, Elementary Schools, N. Y. C. Board of Education. Paper. 26.5×38 cm. 1959. Capitol Publishing Company, Inc., 737 Broadway, New York 3, N. Y. Price \$2.95.

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#### CASTOR OIL CAN BE USED TO MAKE PLASTIC FOAMS

Castor oil can be used as a major ingredient in making plastic foams, the Department of Agriculture reported. Plastic, or urethane, foams made with castor oil range from spongy to rigid, depending on the amount of the oil or other ingredients used and on the methods of processing.

USDA's research with castor oil and plastic foam is part of a broad study designed to discover economic uses for castor beans. Although most of the 120,000,000 pounds of castor oil used in this country each year is imported, interest in domestic production of castor beans is growing.

A market is already established for castor oil as a raw material in jet-engine and other industrial lubricants, in cosmetics and in medicines, the USDA reported.

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#### IMPORT GERMAN BEETLES TO SAVE FIR FORESTS

Beetles from Baden may help control a forest-destroying insect in the U. S. Approximately 20,000 beetles known to prey on the balsam woolly aphid are being imported from West Germany—mostly the Baden area near the city of Freiburg. Live beetles are being released by U. S. Forest Service entomologists on the mostly heavily infested trees in a small control test in Maine.

Some of the imported beetles will also be used to establish a colony in North Carolina. Severe losses of fir trees as a result of aphid infestation have already been reported in Washington, Oregon, Maine and North Carolina.

The balsam woolly aphid damages trees by attaching itself to the trunk and feeding on the sap. Mass infestations injure the trees so severely, the U. S. Department of Agriculture here reported, that trees die in about two years.

Aerial sprays of insecticides do not reach the aphids and ground spraying is too costly. Researches are hopeful that using insect predators will solve the problem.

The Canadian department of agriculture has donated 200,000 tiny flies, scientifically known as *Aphidoletes thompsoni*, that also prey on the aphid. The Canadians have been very successful in their work with both the flies and the beetles (*Laricobius erichsonii*) which they imported, released and colonized in the New Brunswick area.

The British Commonwealth Institute of Biological Control is assisting the USDA's Agricultural Research Service and Forest Service in their aphid control plans.

## Book Reviews

LABORATORY PERFORMANCE TESTS FOR GENERAL PHYSICS, by Haym Kruglak, *Western Michigan University*, and Clifford N. Wall, *University of Minnesota*. Paper. 165 Pages. 15.5×23.5 cm. 1959. Western Michigan University, Kalamazoo, Michigan.

Within the past few years there have been numerous reports concerning needed research in the field of science education. Without exception, all of them have suggested that research studies dealing with the evaluation of the outcomes of science instruction are greatly needed. This publication is one of the efforts designed to meet that need. It is salutary to note that the National Science Foundation supported the efforts of these researchers in undertaking such an important effort.

Specifically, the publication deals with the test items designed to measure outcomes of laboratory performance rather than verbal and theoretical acquisition. The authors point out that these are pilot efforts and "first steps in the development of an adequate testing aid for laboratory work." The task is, therefore, approached honestly and from an objective point of view.

The authors develop the design for the items by first defining the instructional skills and selecting the performance tasks considered to represent the instructional objectives of the laboratory program. The manner in which the trial items and scorings were constructed and the steps taken in the trial administration and test and item analysis are clearly outlined. Finally, the manner in which the items were classified as revised is indicated.

The "final" performance test, together with instructions for administering the test take up the major portion of the document. The students in general are moved from location to location in the physics laboratory, at each of which they are expected to solve a laboratory problem. Two typical problems are these:

"Time: 12 minutes

Apparatus: Spiral spring of known mass; supporting stand; weights; weight holder; stop watch.

Problem: For a spiral spring undergoing oscillations the period is given by

$$T = 2\pi\sqrt{m/k},$$

where  $m$  includes the mass of the weights and the weight hanger and  $\frac{1}{3}$  the mass of the spring, and  $k$  is the force constant of the spring.

Determine  $k$  (dynes/cm) for the spiral spring. The mass of the spring is 172 g.

Note: Record all measurements and computations."

"Apparatus: Assorted apparatus numbered 1 to 16."

Problem: Write down the numbers of those pieces of equipment that you would need to determine the electrical equivalent of heat. (Joule equivalent).

Do not list equipment that is unessential.

\* Needed equipment: calorimeter cup

water  
ammeter and voltmeter  
rheostat  
thermometer  
dropping resistor (optional)  
water heating element  
balance  
line power cord  
clock"

It is pointed out that a number of items were taken from studies undertaken previously by the author and some have not been tested. These points tend to substantiate further the tentative nature of the document. Items are included

from areas such as mechanics, heat, electricity and magnetism, wave motion and sound, optics and contemporary physics.

There is no question about the value of the sample items for the physics teacher who is seeking to broaden his program of evaluation. They probably constitute a pool unobtainable elsewhere.

The reviewer does believe, however, that the report would have benefited from more careful editing. There are a number of grammatical errors and much evidence of awkward construction. For example, the first sentence on page 13 reads, "Furthermore, the presence and use of apparatus in a laboratory best adds a new element or dimension to the test which *naturally* belongs there but which is seldom included." The statement is no less obtuse in content.

Despite these weaknesses, the publication is to be recommended highly and the authors commended.

G.G.M.

ORGANIC CHEMISTRY, Second Edition, A Brief Course, by Ray O. Brewster and William E. McEwen, *Professor of Chemistry*. Cloth. Pages xii+401. 14.5×23 cm. 1959. Prentice-Hall, Inc., Englewood Cliffs, N. J. Price \$7.50.

This textbook of organic chemistry is a revision of a text which first appeared in 1948. At that time, the book was prepared in two parts: the first part treated only the aliphatic compounds and the second part dealt with the aromatics. The second edition uses the unified treatment which the authors feel "is advantageous in a course limited to one semester or two quarters."

In order to gain the interest and attention of the student, the authors have made use of a plan formerly employed by Dr. J. B. Conant in his organic texts in that the first chapter deals with simple alcohols. These being liquids are more tangible to the student than are a series of gaseous hydrocarbons which would be the case if the authors had followed the usual procedure of starting with the alkanes.

In the preface, the authors state: "Throughout the book somewhat more stress has been laid upon mechanisms of reactions than is found in most texts designed for a short course." They later state: "Both the observed facts and their interpretation are important." The reviewer found that the theory is gradually worked into the course as it is needed. For example, the resonance concept is first introduced on Page 64 and later enlarged upon in later chapters, for example, in explaining the properties of nitrobenzene on Page 258 and in aniline on Page 265. The electronic theory is utilized widely in order to explain the properties of various organic compounds.

The list of topics covered is fairly complete and compares well with books planned for a year's course. The subjects of fats, carbohydrates and proteins are given adequate treatment to provide a suitable background for biochemistry. Hence, this textbook should be especially good for a premedical course in organic chemistry.

The publishers have done a commendable job: paper is good, type is easily read; illustrations are well done. A set of questions and problems is found at the end of each chapter.

A good solid course of organic chemistry could easily be developed around this textbook.

GERALD OSBORN

BASIC MATHEMATICS FOR HIGH SCHOOLS, by T. W. Thordarson, *State Director of Supervised Study, Fargo, North Dakota* and R. Perry Anderson, *Chairman of Mathematics, Central High School, St. Paul, Minnesota*. Cloth. Pages vii+310, 16×23.5 cm. 1959. Allyn & Bacon, Inc. 2231 South Parkway, Chicago 16, Illinois.

Here is a text that uses the idea of measurement as a thread to tie the principles developed into one package. It appears to be very well done. The text starts appropriately with the nature of number and the ideas which are best developed through concepts of quantity. This development provides the setting



for further work in the fundamental processes. Measurement is considered in chapter 2 and throughout the rest of the text. A comparison of the metric and English system is made. The approximate nature of measure is discussed along with precision and accuracy.

The number system is extended logically from integers through fractions, decimals, signed numbers and general expressions. Applications are made through angle measure and geometric figures of one, two and three dimensions.

There is material on common measures of liquid, weight, space, laboratory and financial transactions for home and business.

The applications emphasize the mathematical principle being used. The entire text is directed toward the understanding of principles. It is well written and presents a good appearance to the student. It would seem this text would be satisfactory to use with most high school students who have not completed the regular algebra-geometry sequence. I believe it could be used on either a freshman or upper class level with equal success.

PHILIP PEAK  
Indiana University

FINITE MATHEMATICAL STRUCTURES, by John G. Kemeny, *Professor of Mathematics, Dartmouth College*, Hazelton Mirkil, *Assistant Professor of Mathematics, Dartmouth College*, J. Laurie Snell, *Associate Professor of Mathematics, Dartmouth College*, and Gerald L. Thomson, *Professor of Mathematics, Ohio Wesleyan University*. Cloth. Pages xi+487. 1959. 15×23 cm. Prentice-Hall, Inc., 70 Fifth Avenue, New York 11, New York. Price \$7.95.

Those who were pleased with "Introduction to Finite Mathematics," authored by three of the authors of this text, will welcome this text. Whereas the earlier book was designed primarily as a freshman mathematics text for students in the biological and social sciences, the authors have planned this book with more emphasis upon the physical sciences, and for students with more mathematical maturity (a year's course in calculus). However, there is a considerable duplication of material presented in the two books, this one and "Introduction to Finite Mathematics."

It is by no means uncommon to find a text with the statement that it is suitable for any one of several courses, possibly with the idea of increasing sales. This is the case here—the authors state it could be used for an intermediate course following the one-variable calculus and preceding the multi-variable calculus. They likewise feel it is suitable for a semester course in probability, or for a one semester course in linear algebra; another possibility is a year course introducing the student (as for example, a prospective secondary school teacher) to several topics in modern mathematics. In the opinion of the reviewer, the authors have been eminently more successful than is usually the case in producing a multi-purpose text.

The book is exceptionally well written, and a scholarly production. Those seeking a large number of applications from the field of physical science or engineering will be disappointed—the exceptions are however unusual (for example, the use of Markov chains in electrical circuits or in models illustrating the diffusion of gases).

Teachers who felt the earlier book to their liking will no doubt be equally pleased with this volume; on the other hand, those finding the earlier text too difficult or otherwise unsuitable may not find this work to their liking. It may be well to point out that the "infinite" situation is not omitted in this text, but the authors generally develop the "finite" situation first—this emphasis may explain a somewhat misleading title.

The reviewer found only a few relatively minor items which bothered him. In example 2 on page 66 it is apparently assumed as a fact that there is no individual in the United States over 6 feet in height and under 10 years of age. On pages 70–71 (examples 1, 3) similar assumptions were qualified by "perhaps" or "presumably." On page 105 (example 4) use is made of the concept of a two-

component column vector which does not appear to be defined until page 205. The reviewer overlooked the definition of a "one-one function." When the term was encountered, reference to the index failed to locate the term (it was properly defined when first introduced).

Certainly any teacher interested in a text involving the "modern" approach to many mathematical topics cannot ignore this book. Even if it does not fit a particular course, it should be available in the college (or secondary school) library for reference.

CECIL B. READ  
*University of Wichita*

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## The Designs of Jack Frost

John Satterly

*University of Toronto, Toronto, Canada*

How often in an English winter on cold mornings has one exclaimed with delight on beholding the beautiful fern-like patterns that Jack Frost has printed on the window-panes of the dwelling house. They are of indescribable beauty excelling in loveliness any tracery of man.

Yet, this winter in Canada, although we have had plenty of frosty weather, I cannot recall seeing at any time these pictures on the window-panes of our home. All that Jack Frost seems to do is to give an even matt surface of condensate. We have not protective storm-windows. Are my observations corroborated by other observers? Is the reason for this difference in effects due to the fact that in a Canadian home the air is ever so much drier? We all know how cold and damp an English home can be in winter compared with our centrally heated houses. Perhaps the beautiful Jack Frost patterns are given as compensation. This question is discussed by W. J. Humphreys in his *Physics of the Air*.

---

## Ice Denser Than Water

John Satterly

*University of Toronto, Toronto, Canada*

Professor A. N. Shaw of the Physics Department, McGill University, Montreal, describes in "A Note on the Formation of Heavy Ice in a Wollaston's Cryophorus"<sup>1</sup> how in a lecture demonstration, with the lower bulb of the cryophorus in a freezing mixture of ice and salt, ice formed as usual in the upper bulb. The lecture over, the cryophorus warmed up and Shaw saw to his surprise that the cake of ice in the upper bulb sank in its surrounding water. Professor A. S. Eve, the Head of the Department, was called in to witness this unusual state of affairs. Professor Shaw had never seen it before nor has he seen it since and, although I myself must have demonstrated the cryophorus scores of times, I have never seen this effect. McGill, however, has had many unusual phenomena and Shaw recalls that twenty years earlier, Professor John Cox,<sup>2</sup> then Head of the Department, had the same experience and that he had as witnesses Professor H. T. Barnes (of iceberg fame) and a young professor by the name of Ernest Rutherford. I expect the same cryophorus has been used over and over again so that there seems nothing wrong with the water; perhaps it is merely an example of a probability law.

<sup>1</sup> A. N. Shaw, *Proc. Roy. Soc. Can.* Series 3, Vol. 18, 1924.

<sup>2</sup> John Cox, *Proc. Roy. Soc. Can.* Vol. 10, 1904.

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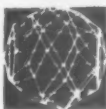
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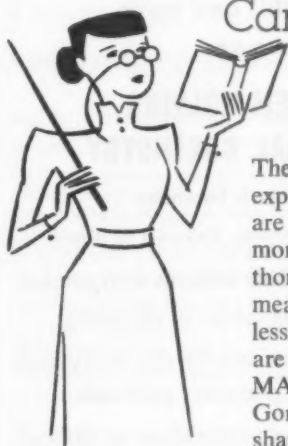
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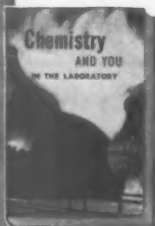
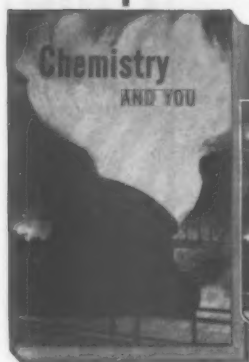
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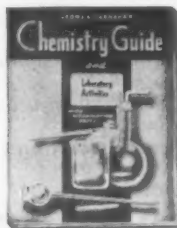
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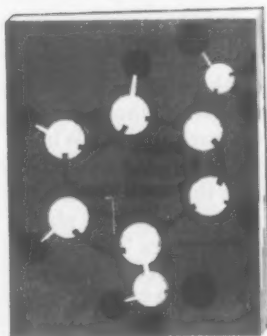


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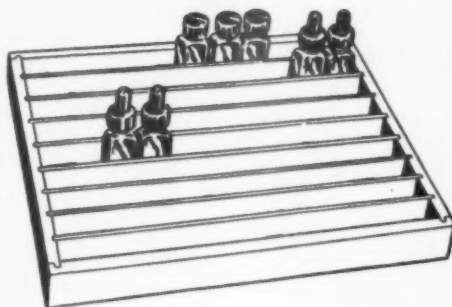
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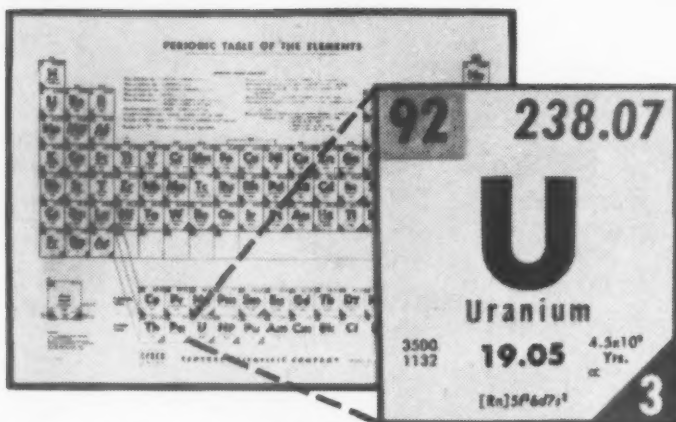
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